Elektrotehnički fakultet
Univerziteta u Sarajevu

PRIPREMNI ZADACI
za
DRUGI PARCIJALNI ISPIT IZ PREDMETA
INŽENJERSKA MATEMATIKA 1
Š.G. 2005 / 2006.

UPUTSTVO:

1. *Za svaki od prva četiri zadatka ponuđena su četiri odgovora od kojih je samo jedan tačan. Za svaki od prva četiri zadatka koje dobijete na ispitu,nakon što ga riješite na ispitu, zaokružite redni broj pod kojim je naveden tačan odgovor za taj zadatak . Zaokruživanje više od jednog odgovora vrednuje se kao i netačan odgovor. Svaki tačan odgovor se boduje sa po* 2,5 *boda, a svaki netačan odgovor se vrednuje sa po* (–0,5) *bodova. Ukoliko se ne zaokruži niti jedan od ponuđena četiri odgovora, za taj zadatak student ostvaruje 0 bodova.*
2. *Peti zadatak, koji dobijete na ispitu, je s otvorenim odgovorom i trebate ga riješiti detaljno. Tačno urađen taj zadatak donosi* 10 *bodova. Boduju se i tačno urađeni dijelovi tog zadatka (pri tom bodovanju najmanja jedinica mjere je* 0,5 *bodova).*

Sarajevo, 10.01. 2006.

Predmetni nastavnik:

____

Doc. Dr. Huse Fatkić

PRIPREMNI ZADACI 1:

Zad.1. Data je familija ($f_\alpha : \alpha \geq 0$) funkcija $f_\alpha : \mathbf{R} \to \mathbf{R}$ definiranih sa

$$f_\alpha(x) := \begin{cases} 2\sqrt{\alpha x^2} - \alpha, & |x| \leq \sqrt{\alpha}, \\ 2x^2 - x^4, & |x| > \sqrt{\alpha}. \end{cases}$$

Ustanovite koja je od sljedeće četiri izjave tačna:

A. Svaka od funkcija f_α $(\alpha \geq 0)$ je neprekidna na **R**.
B. Ni jedna od funkcija f_α $(\alpha \geq 0)$ nije neprekidna na **R.**
C. Postoje dvije vrijednosti od α $(\alpha \geq 0)$ za koje je f_α neprekidna na R.
D. Samo je jedna funkcija f_α iz date familije neprekidna na **R**.

(Ili: Nađite $\lim\limits_{x \to +\infty} \left(\dfrac{x-a}{x-b} \right)^x$. **Rezultat:** e^{b-a}, $(a, b \in \mathbf{R})$.)

Zad.2. Izračunajte (ili ustanovite da ne postoji) sljedeći limes funkcije koristeći asimptotsku relaciju $a^x = 1 + x\ln a + o(x)$ $(x \to 0)$, $(a > 0)$:

$$\lim_{x \to 0} \left(\frac{3^x + 4^x + 5^x}{3} \right)^{\frac{1}{x}}.$$

A. 1.
B. $\sqrt[3]{60}$.
C. 4.
D. Dati limes ne postoji.

(Ili: Napišite *Maclaurinov* razvoj funkcije $f(x) := \sin(\sin x)$ po potencijama od x do člana x^4 sa ostatkom u *Peanovoj* formi. **Rezultat:** Iz $\sin x = x - \frac{1}{6}x^3 + o(x^4)$ slijedi

$$\sin(\sin x) = x - \frac{1}{6}x^3 - \frac{1}{6}\left[x - \frac{1}{6}x^3 + o(x^4)\right]^3 + o(x^4) = x - \frac{1}{6}x^3 - \frac{1}{6}\left[x^3 + o(x^4)\right] + o(x^4) =$$

$$= x - \frac{1}{3}x^3 + o(x^4), (x \to 0).)$$

Zad.3. Izračunajte integrale:

$I := \displaystyle\int \frac{3\sin x + 2\cos x}{2\sin x + 3\cos x} dx$; $J := \displaystyle\int_0^{\frac{\pi}{3}} \ln\left(1 + \sqrt{3}\tan x\right) dx$, (Uputa: Pogodno je koristiti relaciju $\displaystyle\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.).

A. $I = \frac{12}{13}x - \frac{5}{13}\ln|2\sin x + 3\cos x| + C$, $J = \frac{\pi}{3}\ln 2$.

B. $I = \frac{12}{13}x + \frac{5}{13}\ln|2\sin x + 3\cos x| + C$, $J = -\frac{\pi}{2}\ln 2$.

C. $I = \frac{1}{13}\ln\left[e^{12x}\cdot|2\sin x + 3\cos x|^5\right] + C$, $J = \frac{\pi}{3}\ln\sqrt{3}$.

D. $I = \frac{1}{13}\ln\frac{2\cos x - 3\sin x}{2\sin x + 3\cos x} + C$, $J = \frac{\pi}{3}\ln 4$.

Zad.4. Izračunajte limes

$$l:=\lim_{x\to+\infty}\frac{\left(\int_0^x\left(e^{t^2}+10\right)dt\right)^2}{\int_0^x e^{2t^2}dt}.$$

A. $l = +\infty$.
B. $l = 1$.
C. $l = 0$.
D. $l = 5$.

(Ili: Primjenom *Lagrangeove* teoreme (o srednjoj vrijednosti) diferencijalnog računa nađite približnu vrijednost, tačnu na četiri decimale, od arctg 1,003567.

Rezultat: Iz arctg($x+h$)=arctg$x + h\cdot\frac{1}{1+(x+\theta h)^2}$, za $x = 1$ i $h = 0{,}003567$,

slijedi arctg 1,003567 $\begin{cases} <\frac{\pi}{4}+\frac{1}{2}\cdot 0{,}003567 & \text{za } \theta=0,\\ >\frac{\pi}{4}+0{,}003567\cdot\frac{1}{1+1{,}003567} & \text{za } \theta=1,\end{cases}$,tj. 0,787175<arctg 1,003567 <

< 0,787181, pa približna vrijednost od arctg 1,003567, tačna na četiri decimale, iznosi 0,7871.)

Zad.5. Data je realna funkcija f jedne realne promjenljive sa:

$$f(x):=\frac{\left(x^2-2x-15\right)e^x}{\left|x^2-1\right|}\quad (\text{ ili } f(x):=\sqrt[3]{x^4-2x^3+x^2}\ \text{ ili }\ f(x):=(x-a)\,e^{\frac{1}{x}}\ ).$$

a) Odredite prirodni domen Dom (f) i ispitajte ponašanje date funkcije f na rubovima područja Dom (f).

b) Odredite eventualne presjeke grafika date funkcije f sa koordinatnim osama O_x i O_y, a zatim ispitajte znak od f.

c) Odredite eventualne horizontalne, vertikalne i kose asimptote date funkcije f.

d) Nađite ekstreme i tablicu monotonosti za funkciju g datu sa $g(x):=\left|x^2-1\right|\cdot f(x)$.

e) Nađite tačke infleksije i tablicu konveksnosti i konkavnosti za funkciju g datu u d).

f) Odredite sliku Im(g) i nacrtajte grafik funkcije g iz d).

PRIPREMNI ZADACI 2:

Zad.1. Odredite red beskonačno male veličine $\alpha(x) := \mathrm{tg}x - \sin x$ u odnosu na x kad $x \to 0$ i napišite odgovarajuću asimptotsku relaciju.

A. $\alpha(x)$ je beskonačno mala veličina trećeg reda u odnosu na x kad $x \to 0$ i vrijedi asimptotska relacija $\alpha(x) \sim \frac{1}{2}x^3$ $(x \to 0)$.

B. $\alpha(x)$ je beskonačno mala veličina drugog reda u odnosu na x kad $x \to 0$ i vrijedi asimptotska relacija $\alpha(x) \sim \frac{1}{2}x^2$ $(x \to 0)$.

C. $\alpha(x)$ je beskonačno mala veličina istog reda kao i x u odnosu na x kad $x \to 0$ i vrijedi asimptotska relacija $\alpha(x) \sim x$ $(x \to 0)$.

D. $\alpha(x)$ je beskonačno mala veličina trećeg reda u odnosu na x kad $x \to 0$ i vrijedi asimptotska relacija $\alpha(x) \sim \frac{1}{3}x^3$ $(x \to 0)$.

(Ili: Izračunajte limes $l := \lim\limits_{x\to 0} \frac{(1+x)^{10}-1}{x}$. **Rezultat:** $l = 10$.)

Zad.2. Odredite minimalnu površinu trougla ABC čiji je vrh A(-1,0), vrh B je dirালište tangente krive date sa $y = \frac{1}{\sqrt{x}}$, a vrh C je sjecište te tangente sa osi O_x.

A. $\frac{1}{2}\sqrt{3}$.

B. $\frac{1}{2\sqrt{3}}$.

C. $\sqrt{3}$.

D. $2\sqrt{3}$.

(Ili: Funkciju f datu izrazom $f(x) = x^2 \ln^2 x$ u okolini $U(1)$ tačke 1 aproksimirajte *Taylorovim* polinomom četvrtog stepena i procijenite grešku aproksimacije za $x \in \left[\frac{9}{10}, \frac{11}{10}\right]$. **Rezultat:** $f(x) = x^2 \ln^2 x \approx T_4(x) = (x-1)^2 + (x-1)^3 - \frac{1}{12}(x-1)^4$ za svaki $x \in U(1)$, gdje se za $U(1)$ može uzeti skup $(0,+\infty)$; Iz $R_4(x) =$
$$8 \cdot \frac{(x-1)^5}{5!} \cdot \frac{\ln(1+\theta(x-1))}{(1+\theta(x-1))^3}$$
za $x \in \left[\frac{9}{10}, \frac{11}{10}\right]$ slijedi $|R_4(x)| < \frac{1}{15} \cdot \frac{1}{10^5} \cdot \ln\left(1+\frac{1}{10}\right) \approx \frac{1}{15} \cdot \frac{1}{10^6} \approx 6,\dot{6} \cdot 10^{-8}$, jer je $\ln(1+x) \approx x$ za male vrijednosti argumenta x.)

Zad.3. Izračunajte integrale

$$I := \int \left(\frac{1}{\sin x} + \frac{1}{\sinh x} \right) dx\,, \quad J := \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x| \sqrt{\cos x}\, dx\,.$$

A. $I = \ln \left| \sin x \tanh \frac{x}{2} \right| + C\,,\ J = \frac{2}{3}$.

B. $I = \ln \left| \tan \frac{x}{2} \tanh \frac{x}{2} \right| + C\,,\ J = \frac{4}{3}$.

C. $I = \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) \tanh \frac{x}{2} \right| + C\,,\ J = \frac{2}{3}\pi$.

D. $I = \frac{1}{2} \left[\ln \left| \tan \frac{x}{2} \right| + \ln \left| \tanh \frac{x}{2} \right| \right] + C\,,\ J = \frac{4}{3}\pi$.

(Ili: Izračunajte integrale $I := \int \frac{dx}{x^3 + x^2 + x + 1}$, $J := \int_0^{\infty} x^3 e^{-x^2} dx$. **Rezultat:** $I = \frac{1}{4} \ln \frac{(x+1)^2}{x^2+1} + \frac{1}{2} \arctan x + C\,,\ J = \frac{1}{2}$.)

Zad.4. Izračunajte derivaciju funkcije

$$f(x) := \int_{-x}^{x} \exp\left(-\frac{10}{t^2} \right) dt$$

u tački $x = 2$.

A. $f'(2) = \frac{2}{\sqrt[4]{e}}$.

B. $f'(2) = \frac{2}{\sqrt{e^5}}$.

C. $f'(2) = \frac{20}{\sqrt[4]{e}}$.

D. $f'(2) = \frac{10}{\sqrt{e^5}}$.

(Ili: Nađite najmanju i najveću vrijednost funkcije $F(x) := \int_0^x \frac{t-1}{t^2+1} dt$ na segmentu $\left[0, \sqrt{3}\right]$. **Rezultat:** $\min F = \min\left\{F(0), F(1), F\left(\sqrt{3}\right)\right\} = F(1) = \frac{1}{2}\ln 2 - \frac{\pi}{4}$, $\max F = F(0) = 0$.)

Zad.5. Realna funkcija f jedne realne promjenljive definisana je izrazom
$f(x) := \operatorname{sgn}(x)\cdot\ln\left(x+\sqrt{1-x^2}\right)$ (ili $f(x) := \frac{x}{4}+\arcsin\frac{2x}{1+x^2}$ ili $f(x) := \arctan\frac{x^2+1}{x^2-1}$).

g) Odredite prirodni domen $\mathrm{Dom}(f)$ date funkcije f.
h) Odredite eventualne presjeke grafika $G(f)$ funkcije f sa koordinatnim osama.
i) Ispitajte znak date funkcije f.
j) Odredite eventualne tačke prekida i singulariteta i klasificirajte ih za datu funkciju f i njenu recipročnu funkciju $\frac{1}{f}$.
k) Odredite intervale monotonosti i eventualne tačke lokalnog i apsolutnog ekstrema date funkcije f, kao i eventualne prelomne i povratne tačke.
l) Ispitajte konveksnost i konkavnost i odredite eventualne prevojne tačke date funkcije f.
m) Odredite sliku $\mathrm{Im}(f)$ i nacrtajte grafik date funkcije f.

♫

PRENNI ZADACI 1. NIRMEĆ VANJA

$f_\alpha : \alpha \geq 0$. $f_\alpha(x) = \begin{cases} 2\sqrt{\alpha x^2} - \alpha & |x| \leq \sqrt{\alpha} \\ 2x^2 - x^4, & |x| > \sqrt{\alpha} \end{cases}$

F-JA $f_\alpha(x)$ JE DEFINISANA I NEPREKIDNA KAO ELEMENTARNA $\forall x \neq \pm\sqrt{\alpha}$, V_1 TAČKE U OSIMA ĆEMO ISPITIVATI NEPREKIDNOST SU $x_1 = -\sqrt{\alpha}$, $x_2 = \sqrt{\alpha}$

$x_1 = -\sqrt{\alpha}$

$\lim\limits_{x \to -\sqrt{\alpha}_-} f_\alpha(x) = \lim\limits_{x \to -\sqrt{\alpha}} (2x^2 - x^4) = 2\alpha - \alpha^2$

$\lim\limits_{x \to -\sqrt{\alpha}_+} f_\alpha(x) = \lim\limits_{x \to -\sqrt{\alpha}_+} (2\sqrt{\alpha x^2} - \alpha) = \alpha$

$f(-\sqrt{\alpha}) = \alpha$

DA BI $f_\alpha(x)$ BILA NEPREKIDNA U TAČKI $x = -\sqrt{\alpha}$. POTREBNO JE DA VRIJEDI $2\alpha - \alpha^2 = \alpha \Rightarrow \alpha^2 - \alpha = 0 \Rightarrow \boxed{\alpha_1 = 0, \ \alpha_2 = 1}$ (1)

$x_2 = \sqrt{\alpha}$

$\lim\limits_{x \to \sqrt{\alpha}_-} f_\alpha(x) = \lim\limits_{x \to \sqrt{\alpha}_-} (2\sqrt{\alpha x^2} - \alpha) = \alpha$

$\lim\limits_{x \to \sqrt{\alpha}_+} f_\alpha(x) = \lim\limits_{x \to \sqrt{\alpha}_+} (2x^2 - x^4) = 2\alpha - \alpha^2$

$f(\sqrt{\alpha}) = \alpha$

ZA NEPREKIDNOST JE POTREBNO

$\Rightarrow 2\alpha - \alpha^2 = \alpha \Rightarrow \boxed{\alpha_1 = 0, \ \alpha_2 = 1}$ (2)

ZAKLJUČUJEMO IZ (1) I (2) DA POSTOJE 2 VRIJEDNOSTI ZA α, $\alpha_1 = 0$ I $\alpha_2 = 1$, ZA KOJE JE F-JA $f_\alpha(x)$ NEPREKIDNA $\forall x \in \mathbb{R}$, TJ NA SKUPU $\mathbb{R}$.

2) $\lim\limits_{x \to +\infty} \left(\frac{x-a}{x-b}\right)^x = \lim\limits_{x \to \infty} \left(\frac{1 - \frac{a}{x}}{1 - \frac{b}{x}}\right)^x = \frac{\lim\limits_{x \to \infty} (1 - \frac{a}{x})^x}{\lim\limits_{x \to \infty} (1 - \frac{b}{x})^x} = \frac{e^{-a}}{e^{-b}} = e^{b-a}$.

$a, b \in \mathbb{R}$, $a \neq b$

$) \; a^x = 1 + x\ln a + o(x) \quad (x\to 0)\,(a>0)$

$$\lim_{x\to 0}\left(\frac{3^x+4^x+5^x}{3}\right)^{\frac{1}{x}} = \lim_{x\to 0}\left[\frac{1+x\ln 3+o(x)+1+x\ln 4+o(x)+1+x\ln 5+o(x)}{3}\right]$$

$$=\lim_{x\to 0}\left[\frac{3+x(\ln 3+\ln 4+\ln 5)}{3}\right]^{\frac{1}{x}} = \left|\begin{matrix} x=\frac{1}{t} \\ x\to 0 \\ t\to\infty\end{matrix}\right| = \lim_{t\to\infty}\left[1+\frac{\ln 60}{3t}\right]^t$$

$$=\lim_{t\to\infty}\left(1+\frac{\ln\sqrt[3]{60}}{t}\right)^t = \boxed{\ln\sqrt[3]{60}}$$

$*) \; f(x) = \sin(\sin x)$

$$\sin x = x - \frac{1}{6}x^3 + o(x^4)$$

$$\sin(\sin x) = x - \frac{1}{6}x^3 - \frac{1}{6}\left[x-\frac{1}{6}x^3+o(x^4)\right]^3 + o(x^4) = x - \frac{1}{6}x^3 - \frac{1}{6}\left[x^3+o(x^4)\right] + o(x^4$$

$$= x - \frac{1}{3}x^3 + o(x^4), \quad x\to 0$$

$$I = \int \frac{3\sin x + 2\cos x}{2\sin x + 3\cos x}dx = \int \frac{3tg\,x + 2}{2tg\,x + 3}dx = \left| \begin{array}{l} tg\,x = t \\ \frac{dx}{\cos^2 x} = dt \\ dx = \frac{dt}{t^2+1} \end{array} \right.$$

$\sin^2 x + \cos^2 x = 1$

$tg^2 x + 1 = \frac{1}{\cos^2 x}$

$t^2 + 1 = \frac{1}{\cos^2 x}$

$$I = \int \frac{3t+2}{2t+3} \cdot \frac{dt}{t^2+1}$$

RASTAVLJANJE NA PARCIJALNE RAZLOMKE

$$\frac{3t+2}{(2t+3)(t^2+1)} = \frac{A}{2t+3} + \frac{Bt+C}{t^2+1} \Rightarrow 3t+2 = A(t^2+1) + (Bt+C) \cdot 2t+3;$$

$$3t+2 = At^2 + A + 2Bt^2 + 3Bt + 2Ct + 3C$$

$A + 2B = 0$ $\quad -2B + 3C = 2 \;/\cdot 3$

$3B + 2C = 3$ $\quad 3B + 2C = 3 \;/\cdot 2$ $\quad \Rightarrow 13C = 12$

$A + 3C = 2$ $\quad C = \frac{12}{13}$

$A = 2 - 3C = -\frac{10}{13}$

$B = -\frac{A}{2} = \frac{5}{13}$

$$I = -\frac{10}{13}\int \frac{dt}{2t+3} + \frac{1}{13}\int \frac{5t+12}{t^2+1}dt$$

$$I = -\frac{10}{13}\int \frac{dt}{2t+3} + \frac{5}{13}\int \frac{t\,dt}{t^2+1} + \frac{12}{13}\int \frac{dt}{t^2+1}$$

$$I = -\frac{10}{13} \cdot \frac{1}{2}\ln|2t+3| + \frac{5}{13} \cdot \frac{1}{2}\ln|t^2+1| + \frac{12}{13} \cdot arctg\,t + C \quad \Big| x = arctg\,t$$

$$I = -\frac{5}{13} \cdot \ln\left(\frac{|2t+3|}{\sqrt{t^2+1}}\right) + \frac{12}{13} \cdot arctg\,t + C$$

$$= -\frac{5}{13} \cdot \ln\left(\frac{|2tg\,x+3|}{\sqrt{tg^2 x+1}}\right) + \frac{12}{3} \cdot x + C = -\frac{5}{13} \cdot \ln|2\sin x + 3\cos x| + \frac{12}{13}x + C$$

$$J = \int_0^{\pi/3} \ln(1+\sqrt{3}\,\mathrm{tg}\,x)\,dx$$

релацијом $$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$J = \int_0^{\pi/3} \ln\left[1+\sqrt{3}\,\mathrm{tg}\left(\frac{\pi}{3}-x\right)\right] dx$$

$$\mathrm{tg}\left(\frac{\pi}{3}-x\right) = \frac{\mathrm{tg}\frac{\pi}{3} - \mathrm{tg}\,x}{1+\mathrm{tg}\frac{\pi}{3}\,\mathrm{tg}\,x} = \frac{\sqrt{3}-\mathrm{tg}\,x}{1+\sqrt{3}\,\mathrm{tg}\,x}$$

$$J = \int_0^{\pi/3} \ln\left[1+\sqrt{3}\cdot\frac{\sqrt{3}-\mathrm{tg}\,x}{1+\sqrt{3}\,\mathrm{tg}\,x}\right] dx = \int_0^{\pi/3} \ln\left[\frac{1+\cancel{\sqrt{3}\,\mathrm{tg}\,x}+3-\cancel{\sqrt{3}\,\mathrm{tg}\,x}}{1+\sqrt{3}\,\mathrm{tg}\,x}\right] dx$$

$$J = \int_0^{\pi/3} \ln\frac{4}{1+\sqrt{3}\,\mathrm{tg}\,x}\,dx = \int_0^{\pi/3} \ln 4\,dx - \int_0^{\pi/3} \ln(1+\sqrt{3}\,\mathrm{tg}\,x)\,dx = \ln 4\cdot x\Big|_0^{\pi/3} - J$$

$$2J = \ln 4\cdot\frac{\pi}{3} \Rightarrow 2J = 2\ln 2\cdot\frac{\pi}{3} \Rightarrow \underline{J = \frac{\pi}{3}\ln 2}$$

$$l=\lim_{x\to\infty}\frac{\left[\int_0^x (e^{t^2}+10)\,dt\right]^2}{\int_0^x e^{2t^2}\,dt}$$

PODINTEGRALNE F-JE $f_1(t)=e^{t^2}+10$ I $f_2(t)=e^{2t^2}$ SU POZITIVNE I STROGO RASTUĆE, A KAKO JE $x>0$ ⇒ TO INTEGRALI $J_1\to\infty$, $J_2\to\infty$ PA IMAMO NEODREĐENI OBLIK LIMESA $\frac{\infty}{\infty}$ ⇒ KORISTIMO LOPITALOVO PRAVILO

$$=\lim_{x\to\infty}\frac{\frac{d}{dx}\left\{\left[\int_0^x (e^{t^2}+10)\,dt\right]^2\right\}}{\frac{d}{dx}\left(\int_0^x e^{2t^2}\,dt\right)}.$$

KAKO SU F-JE PREDSTAVLJENE PREKO INTEGRALA DIFERENCIRANJE VRŠIMO PREKO FORMULE

$$\frac{d}{dx}\left[\int_{f_2(x)}^{f_1(x)} f(x)\,dx\right]=f(f_1(x))\cdot\frac{df_1(x)}{dx}-f(f_2(x))\cdot\frac{df_2(x)}{dx}$$

$$=\lim_{x\to\infty}\frac{2\cdot\int_0^x (e^{t^2}+10)\,dt\cdot(e^{x^2}+10)}{e^{2x^2}}\overset{\frac{\infty}{\infty}}{=}\lim_{x\to\infty}\frac{2\cdot(e^{x^2}+10)\cdot(e^{x^2}+10)+2\cdot\int_0^x (e^{t^2}+10)\,dt\cdot e^{x^2}\cdot 2x}{e^{2x^2}\cdot 4x}$$

$$=\lim_{x\to\infty}\frac{(e^{x^2}+10)^2+2e^{x^2}\cdot x\cdot\int_0^x (e^{t^2}+10)\,dt}{2xe^{2x^2}}=\lim_{x\to\infty}\frac{e^{2x^2}+20e^{x^2}+100}{2xe^{2x^2}}+$$

$$\lim_{x\to\infty}\frac{\cancel{2}e^{x^2}\cdot\cancel{x}\cdot\int_0^x (e^{t^2}+10)\,dt}{\cancel{2x}e^{2x^2}}=\lim_{x\to\infty}\frac{1}{2x}+\lim_{x\to\infty}\frac{10}{xe^{x^2}}+\lim_{x\to\infty}\frac{100}{2xe^{2x^2}}$$

LOPITAL

$$\lim_{x\to\infty}\frac{\int_0^x (e^{t^2}+10)\,dt}{e^{x^2}}=0+0+0+\lim_{x\to\infty}\frac{e^{x^2}+10}{e^{x^2}\cdot 2x}=\lim_{x\to\infty}\frac{1}{2x}+\lim_{x\to\infty}\frac{10}{e^{x^2}\cdot 2x}=0$$

)

$$\frac{f(b)-f(a)}{b-a}=f'(\xi)\qquad \xi=a+\theta\cdot(b-a)\qquad 0<\theta<1$$

$f(x)=\operatorname{arctg} x$, DEFINISANA NEPREKIDNA. $\forall x$

$f'(x)=\frac{1}{x^2+1}$ $\exists, \forall x$

$a=1$ $b=1{,}002567$ $b-a=h$ $f(a+h)=f'(a+\theta\cdot h)\cdot h+f(a)$

$$\operatorname{arctg}(a+h) = \operatorname{arctg} a + \frac{h}{1+(a+\Theta h)^2} \quad (*)$$

$0 < \Theta < 1$

$0 < \Theta h < h \quad / + a$

$a < a + \Theta h < a + h$

$a^2 < (a+\Theta h)^2 < (a+h)^2 \quad / + 1$

$1 + a^2 < 1 + (a+\Theta h)^2 < 1 + (a+h)^2$

$$\frac{1}{1+(a+h)^2} < \frac{1}{1+(a+\Theta h)^2} < \frac{1}{1+a^2} \Rightarrow \frac{h}{1+(a+h)^2} < \frac{h}{1+(a+\Theta h)^2} < \frac{h}{1+a^2}$$

$$\Downarrow$$

$$\frac{0{,}003567}{1+(1{,}003567)^2} < \frac{h}{1+(a+\Theta h)^2} < \frac{0{,}003567}{1+1}$$

$$1{,}777149602 \cdot 10^{-3} < \frac{h}{1+(\Theta h)^2} < 1{,}7835 \cdot 10^{-3}$$

SADA IZ $(*)$ SLIJEDI

$$\frac{\pi}{4} + 1{,}777149602 \cdot 10^{-3} < \operatorname{arctg}(1{,}003567) < \frac{\pi}{4} + 1{,}7835 \cdot 10^{-3}$$

$$0{,}787175313 < \operatorname{arctg}(1{,}003567) < 0{,}787181663$$

PA JE TAČNA VRIJEDNOST NA 4 DECIMALE $\operatorname{arctg}(1{,}003567) = \underline{\underline{0{,}7871}}$

$f(x) = \operatorname{arctg} \frac{x^2+1}{x^2-1}$

D.P. $x^2-1 \neq 0 \Rightarrow |x| \neq 1 \Rightarrow$ D.P. $R \setminus \{-1, 1\}$

$x=0 \in D.P \Rightarrow f(0) = \operatorname{arctg}(-1) = -\frac{\pi}{4}$

$y=0 \Leftrightarrow 0 = \operatorname{arctg} \frac{x^2+1}{x^2-1} \Rightarrow \frac{x^2+1}{x^2-1} = 0$ NEMA RJEŠENJA! F-JA NE SIJEČE X-OSU

F-JA $f(x)$ JE PARNA JER JE $f(-x) = f(x) \Rightarrow$ SIMETRIČNA U ODNOSU NA Y-OSU

$f(x) > 0 \Rightarrow \frac{x^2+1}{x^2-1} > 0 \Rightarrow x^2-1 > 0 \Rightarrow |x| > 1$

$f(x) < 0 \Rightarrow \frac{x^2+1}{x^2-1} < 0 \Rightarrow x^2-1 < 0 \Rightarrow |x| < 1$

KRITIČNE TAČKE SU $x = \pm 1$

$\lim\limits_{x\to -1-} \left(\operatorname{arctg} \frac{x^2+1}{x^2-1}\right) = \frac{\pi}{2}$

$\lim\limits_{x\to -1+} \left(\operatorname{arctg} \frac{x^2+1}{x^2-1}\right) = -\frac{\pi}{2}$ PREKID I VRSTE

$\lim\limits_{x\to 1-} \left(\operatorname{arctg} \frac{x^2+1}{x^2-1}\right) = -\frac{\pi}{2}$

$\lim\limits_{x\to 1+} \left(\operatorname{arctg} \frac{x^2+1}{x^2-1}\right) = \frac{\pi}{2}$ PREKID I VRSTE

$\lim\limits_{x\to \pm\infty} \left(\operatorname{arctg} \frac{x^2+1}{x^2-1}\right) = \lim\limits_{x\to\pm\infty}\left(\operatorname{arctg} \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}\right) = \frac{\pi}{4}$ H.A. $x = \frac{\pi}{4}$

VERTIKALNIH ASIMPTOTA NEMA!, KOSIH NEMA!

F-JA $g(x) = \frac{1}{f(x)} = \frac{1}{\operatorname{arctg}\frac{x^2+1}{x^2-1}}$

$\Rightarrow \lim\limits_{x\to -1-} g(x) = \frac{2}{\pi}$, $\lim\limits_{x\to -1+} g(x) = -\frac{2}{\pi}$ — I VRSTE PREKID

$\lim\limits_{x\to 1-} g(x) = -\frac{2}{\pi}$, $\lim\limits_{x\to 1+} g(x) = \frac{2}{\pi}$ — I VRSTE PREKID

$\lim\limits_{x\to\pm\infty} g(x) = \frac{4}{\pi}$

$f'(x) = \frac{1}{1+\left(\frac{x^2+1}{x^2-1}\right)^2} \cdot \left(\frac{x^2+1}{x^2-1}\right)' = \frac{\cancel{(x^2-1)^2}}{(x^2-1)^2+(x^2+1)^2} \cdot \frac{2x(x^2-1)-2x(x^2+1)}{\cancel{(x^2-1)^2}} = \frac{-4x}{x^4-2x^2+1+x^4+2x^2+1} = \frac{-2x}{x^4+1}$

$f'(x) = \frac{-2x}{x^4+1} \Rightarrow$

$f'(x) > 0 \Rightarrow \frac{-2x}{x^4+1} > 0 \Rightarrow -2x > 0 \Rightarrow x < 0$, $f(x) \nearrow$

$f'(x) < 0 \Rightarrow \frac{-2x}{x^4+1} < 0 \Rightarrow -2x < 0 \Rightarrow x > 0$, $f(x) \searrow$

$f'(x) = 0 \Rightarrow x_s = 0$

$$f''(x) = \left(\frac{-2x}{x^4+1}\right)' = \frac{-2(x^4+1)-(-2x)\cdot 4x^3}{x^4+1} = \frac{6x^4-2}{x^4+1} = \frac{2}{x^4+1}\cdot(3x^4-1)$$

$$f''(x) > 0 \Rightarrow \frac{2}{x^4+1}\cdot(3x^4-1) > 0 \Rightarrow 3x^4-1>0 \Rightarrow (\sqrt{3}x^2-1)(\sqrt{3}x^2+1)>0 \Rightarrow \sqrt{3}x^2-1>0$$

$|x| > \frac{1}{\sqrt[4]{3}}$ ← ∪ KONVEKSNA

$$f''(x) < 0 \Rightarrow \frac{2}{x^4+1}(3x^4-1) < 0 \Rightarrow 3x^4-1<0 \Rightarrow (\sqrt{3}x^2-1)(\sqrt{3}x^2+1)<0 \Rightarrow \sqrt{3}x^2-1<0$$

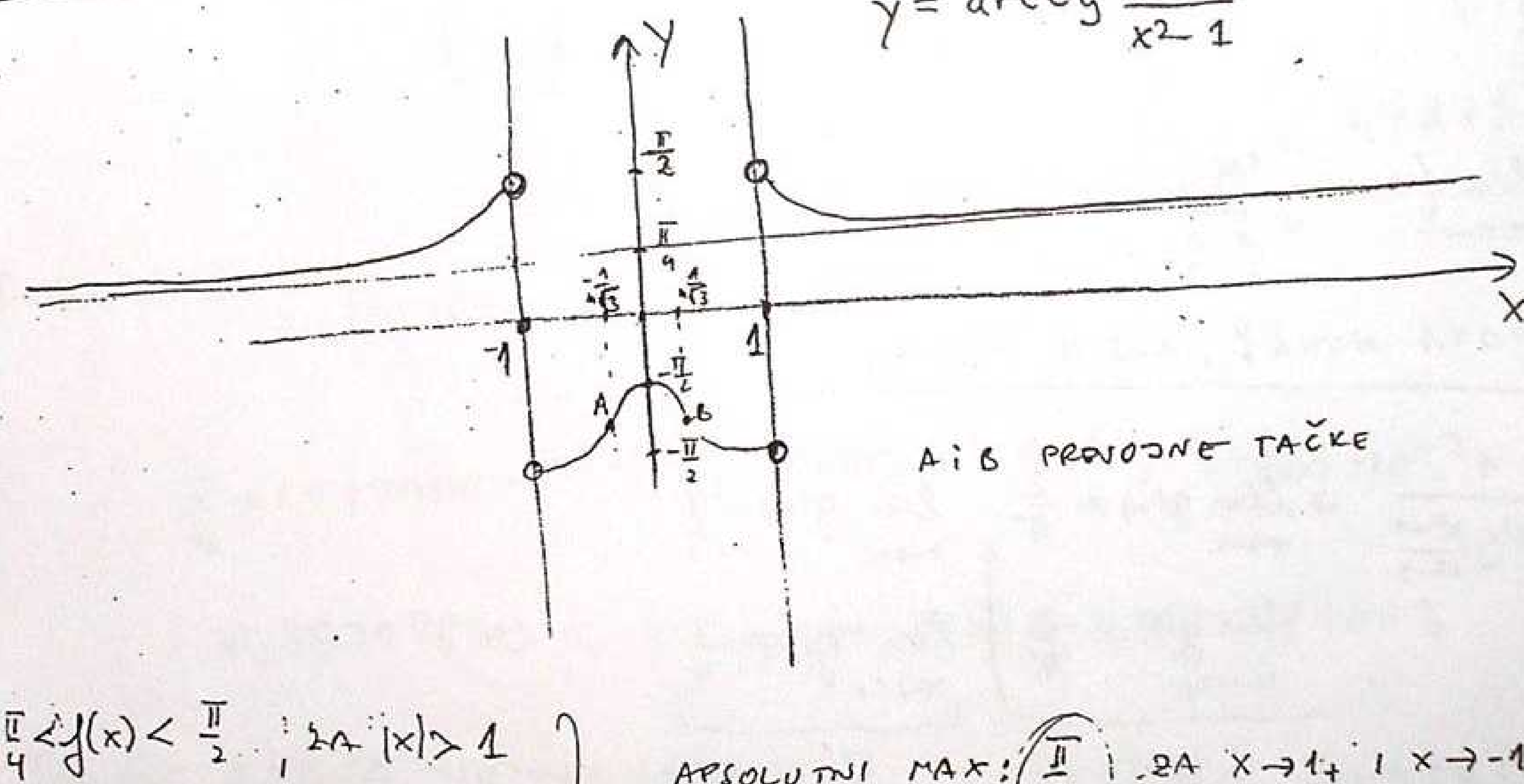
$|x| < \frac{1}{\sqrt[4]{3}}$ ∩ KONKAVNA

$f''(x) = 0 \Rightarrow 3x^4-1=0 \Rightarrow x_1 = \frac{1}{\sqrt[4]{3}}, \; x_2 = -\frac{1}{\sqrt[4]{3}}$ PREVOJNE TAČKE

$x_S = 0$

$f''(0) = -2 < 0 \Rightarrow f(0)$ IMAMO LOKALNI MAXIMUM

$$y = \operatorname{arctg}\frac{x^2+1}{x^2-1}$$



A i B PREVOJNE TAČKE

$\frac{\pi}{4} < f(x) < \frac{\pi}{2}$; ZA $|x| > 1$

$-\frac{\pi}{2} < f(x) < -\frac{\pi}{4}$ ZA $|x| < 1$

APSOLUTNI MAX: $\frac{\pi}{2}$; ZA $x \to 1_+$, $x \to -1_-$

APSOLUTNI MIN: $-\frac{\pi}{2}$ ZA $x \to 1_-$, $x \to -1_+$

LOKALNI MAX: $-\frac{\pi}{4}$ ZA $x = 0$

$$R(f) = \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$f(x) = x^2 \ln^2 x \qquad x \in \left[9/10, 11/10 \right]$

$f(1) = 0$

$f'(x) = 2x \ln^2 x + x^2 \cdot 2\ln x \cdot \frac{1}{x} = 2x\ln^2 x + 2x\ln x \Rightarrow f'(1) = 0$

$f''(x) = 2\ln^2 x + 2\cancel{x} \cdot 2\ln x \cdot \frac{1}{\cancel{x}} + 2\ln x + 2x \cdot \frac{1}{x} = 2\ln^2 x + 6\ln x + 2 \Rightarrow f''(1) = 2$

$f'''(x) = 4\ln x \cdot \frac{1}{x} + 6 \cdot \frac{1}{x} \Rightarrow f'''(x) = 6$

$f^{IV}(x) = (4x^{-1}\ln x)' + (6x^{-1})' = (-4x^{-2}\ln x) + 4x^{-1} \cdot \frac{1}{x} - \frac{6}{x^2} = \frac{-4\ln x - 2}{x^2} \Rightarrow f^{IV}(1) = -2$

$f^{V}(x) = \frac{\left(-\frac{4}{x}\right) \cdot x^2 - (-4\ln x - 2) \cdot 2x}{x^4} = \frac{-4x + 8x\ln x + 4x}{x^4} = \frac{8\ln x}{x^3}$

$f(x) = f(1) + f'(1) \cdot \frac{x-1}{1!} + f''(1) \cdot \frac{(x-1)^2}{2!} + f'''(1) \cdot \frac{(x-1)^2}{3!} + f^{IV}(1) \cdot \frac{(x-1)^4}{4!} + f^{V}(\xi) \cdot \frac{(x-1)^5}{5!}$

$\xi = 1 + \theta(x-1) \quad 0 < \theta < 1$

$f(x) = (x-1)^2 + (x-1)^3 - \frac{(x-1)^4}{12} + \frac{(x-1)^5}{5!} \cdot 8 \, \frac{\ln[1+\theta(x-1)]}{[1+\theta(x-1)]^3}$

$|R_4(x)| < \frac{8}{5!} \cdot \frac{1}{10^5} \ln\left(1 + \frac{1}{10}\right) \approx \frac{1}{15} \cdot \frac{1}{10^6} \approx 6{,}6 \cdot 10^{-8}$

$\ln(1+x) \sim x$, ZA $x <<$

Skinuto sa www.etf.ba

3*) $I = \int \frac{dx}{x^3+x^2+x+1}$ $\quad J = \int_0^\infty x^3 e^{-x^2} dx$

$$I = \int \frac{dx}{(x^2+1)(x+1)} \Rightarrow \frac{1}{(x^2+1)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} \Rightarrow 1 = A(x^2+1) + (Bx+C)(x+1)$$

$$1 = Ax^2 + A + Bx^2 + x(B+C) + C$$

$A+B=0$

$B+C=0$

$A+C=1 \Rightarrow A=C=\frac{1}{2}$

$B=-\frac{1}{2}$

$$I = \frac{1}{2}\int \frac{dx}{x+1} - \frac{1}{2}\int \frac{x-1}{x^2+1}dx \Rightarrow \frac{1}{2}\ln|x+1| - \frac{1}{2}\int \frac{x}{x^2+1}dx + \int \frac{dx}{x^2+1}$$

$$I = \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln|x^2+1| + \operatorname{arctg} x + C$$

$$I = \frac{1}{4}\cdot \ln|x+1|^2 - \frac{1}{4}\ln(x^2+1) + \operatorname{arctg} x + C = \frac{1}{4}\ln\frac{|x+1|^2}{x^2+1} + \operatorname{arctg} x + C$$

$$J = \int_0^\infty x^3 e^{-x^2} dx = \begin{vmatrix} x^2 = t \\ 2x\,dx = dt \\ x_d = 0 \quad t_d = 0 \\ x_g = \infty \quad t_g = \infty \end{vmatrix} = \int_0^\infty \frac{t \cdot e^{-t}}{2} dt = \frac{1}{2}\int_0^\infty t e^{-t} dt$$

$$\int t e^{-t} = (At+B)e^{-t} \Rightarrow te^{-t} = Ae^{-t} + (At+B)\cdot(-e^{-t}) \Rightarrow A-B=0$$

$$A=-1 \Rightarrow B=-1$$

$$J = \frac{1}{2}(-t-1)e^{-t}\Big|_0^\infty = \frac{1}{2}\lim_{t\to\infty}(-t-1)e^{-t} - \frac{1}{2}(-1)e^0 = -\frac{1}{2}\lim_{t\to\infty}\frac{t+1}{e^t} + \frac{1}{2}$$

LOPITAL

$$J = -\frac{1}{2}\lim_{t\to\infty}\cancel{\frac{1}{e^t}}^{\to 0} + \frac{1}{2} = \frac{1}{2}$$

PRIPREMNI ZADACI 2

1)

$$\lim_{x\to 0}\frac{tg\,x-\sin x}{x^n} \qquad tg\,x = x+\frac{x^3}{3}+o(x^4)$$

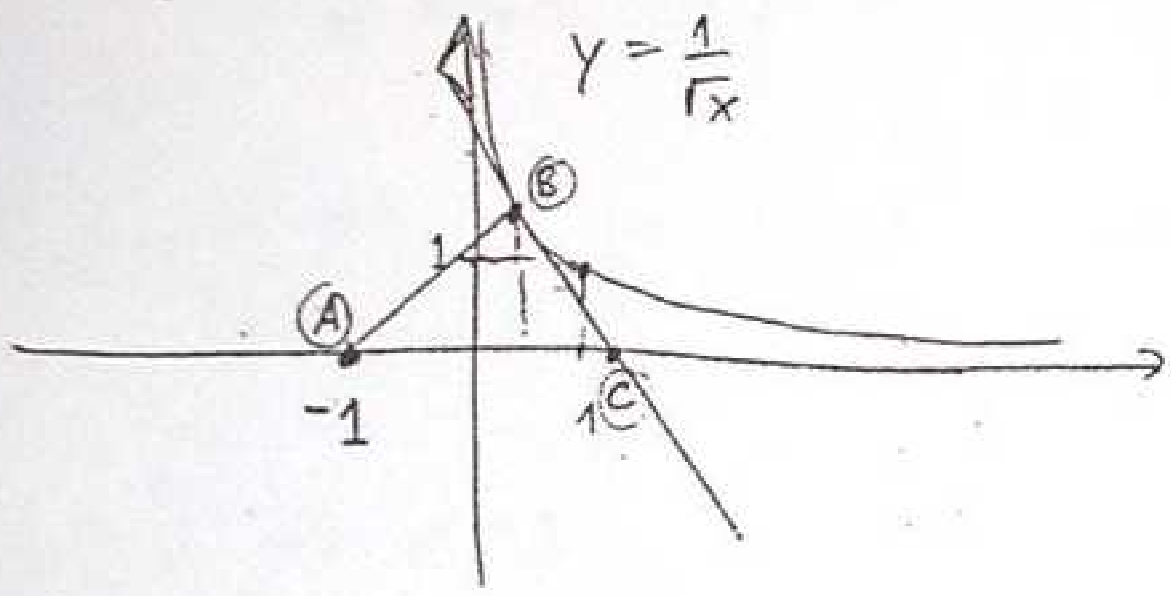
$$\sin x = x-\frac{x^3}{6}+o(x^4)$$

$$tg\,x-\sin x = \frac{1}{2}x^3+o(x^4) \Rightarrow \lim_{x\to 0}\frac{tg\,x-\sin x}{x^3} = \lim_{x\to 0}\frac{\frac{1}{2}x^3+o(x^4)}{x^3}=\frac{1}{2}$$

$$tg\,x-\sin x \sim \frac{1}{2}x^3$$

. BESKONAČNO MALA VELIČINA 3 REDA

2)



$A(-1,0)$

$B(x_B, f(x_B))$

C

PRAVA $\overline{BC}$: $y-y_B=k(x-x_B)$ $\qquad f'(x)=(x^{-\frac{1}{2}})'=-\frac{1}{2}x^{-\frac{3}{2}}$

$$y_B=\frac{1}{\sqrt{x_B}} \qquad k=f'(x_B)$$

$$k=-\frac{1}{2}x_B^{-\frac{3}{2}}$$

$$y-x_B^{-\frac{1}{2}}=-\frac{1}{2}x_B^{-\frac{3}{2}}(x-x_B) \quad \text{PRAVA } \overline{BC}$$

$$y=0, \quad -x_B^{-\frac{1}{2}}=-\frac{1}{2}x_B^{-\frac{3}{2}}(x-x_B) \;/\cdot(-x_B^{\frac{3}{2}})\cdot 2$$

$$2x_B = x_B(x_C-x_B) \Rightarrow 2=x_C-x_B$$

$$x_C=2+x_B$$

$$P_{\Delta ABC}=\frac{AC\cdot h}{2}=\frac{x_C+1}{2}\cdot f(x_B)=\frac{3+x_B}{2}\cdot x_B^{-\frac{1}{2}}$$

$$\frac{dP}{dx_B}=0 \Rightarrow \frac{1}{2}x_B^{-\frac{1}{2}}+\frac{3+x_B}{2}\cdot\left(-\frac{1}{2}\right)x_B^{-\frac{3}{2}}=0 \;/\cdot 4x_B^{\frac{3}{2}}$$

$$2x_B+(3+x_B)\cdot(-1)=0$$

$$2x_B-3-x_B=0 \Rightarrow$$

$$(x_B=3) \rightarrow \boxed{P_{\Delta ABC}=\frac{3+3}{2}\cdot\frac{1}{\sqrt{3}}=\sqrt{3}}$$

*)
$$l = \lim_{x\to 0} \frac{(1+x)^{10}-1}{x} = \lim_{x\to 0} \frac{\left[e^{\ln(1+x)}\right]^{10}-1}{x} = \lim_{x\to 0} \frac{e^{10\ln(1+x)}-1}{x}$$

$\ln(1+x) = t$

$x\to 0 \to t\to 0$

$x = e^t - 1$

$$l = \lim_{t\to 0} \frac{e^{10t}-1}{e^t-1} = \lim_{t\to 0} \underbrace{\frac{e^{10t}-1}{10t}}_{\to 1} \cdot \underbrace{\frac{10t}{e^t-1}}_{\to 1}$$

$$l = \lim_{t\to 0} \underbrace{\frac{e^{10t}-1}{10t}}_{1} \cdot 10 \cdot \lim_{t\to 0} \underbrace{\frac{t}{e^t-1}}_{1} = 10$$

$$f(x)=\int_{-x}^{x} e^{-\frac{10}{t^2}}\,dt \qquad x=2$$

$$f'(x)=\left(e^{-\frac{10}{t^2}}\right)\Big|_{t=x}\cdot\underbrace{\frac{dx}{dx}}_{1}-e^{-\frac{10}{t^2}}\Big|_{t=-x}\cdot\underbrace{\frac{d(-x)}{dx}}_{-1}=e^{-\frac{10}{t^2}}+e^{-\frac{10}{t^2}}=2e^{-\frac{10}{t^2}}$$

$$f'(2)=2\cdot e^{-\frac{10}{4}}=2e^{-\frac{5}{2}}=\frac{2}{\sqrt{e^5}}$$

4*). $$F(x)=\int_0^x \frac{t-1}{t^2+1}\,dt \qquad x\in[0,\sqrt{3}]$$

PODINTEGRALNA F-JA $\frac{t-1}{t^2+1}$ JE NEPREKIDNA NA SEGMENTU $[0,\sqrt{3}]$, PA JE I ~~[illegible]~~ INTEGRABILNA I NA SEGMENTU $[0,\sqrt{3}]$ DOSTIŽE MIN I MAX VRIJEDNOST

$$F'(x)=\left(\frac{t-1}{t^2+1}\right)_{t=x}\cdot\frac{dx}{dx}=\frac{x-1}{x^2+1}$$

$F'(x)>0, \quad x\in(1,\sqrt{3}]$

$F'(x)<0 \quad x\in[0,1)$

$F'(x)=0 \Rightarrow x=1$

POTENCIJALNE TAČKE EKSTREMA SU KRAJNJE TAČKE SEGMENTA $[0,\sqrt{3}]$ I STACIONAR. TAČKA $x=1$

$$F''(x)=\frac{x^2+1-(x-1)\cdot 2x}{(x^2+1)^2}=\frac{-x^2+2x+1}{(x^2+1)^2}$$

$F''(1)=\frac{2}{}>0 \quad F(1)$ MINIMUM

$F(0)=0$

$$F(x)=\int_0^x\frac{t}{t^2+1}\,dt-\int_0^x\frac{dt}{t^2+1}=\frac{1}{2}\ln(t^2+1)\Big|_0^x-\operatorname{arctg}t\Big|_0^x=\frac{1}{2}\ln(x^2+1)-\operatorname{arctg}x$$

$$F(\sqrt{3})=\frac{1}{2}\ln 4-\operatorname{arctg}\sqrt{3}=\ln 2-\frac{\pi}{3}<0$$

$$F(1)=\frac{1}{2}\ln 2-\operatorname{arctg}1=\frac{\ln 2}{2}-\frac{\pi}{4}<0$$

$\min F=F(1)=\frac{1}{2}\ln 2-\frac{\pi}{4}$

$\max F=F(0)=0$

$f(x) = \sqrt[3]{x^4 - 2x^3 + x^2}$

D.P. $x \in \mathbb{R}$.

$f(x) = \sqrt[3]{x^2(x^2 - 2x + 1)} = \sqrt[3]{[x(x-1)]^2}$

$f(x) \geq 0 \quad \forall x \in \mathbb{R}$, $f(x) = 0 \Rightarrow x_1 = 0$, $x_2 = 1$

$\lim_{x \to \pm\infty} \sqrt[3]{[x(x-1)]^2} = +\infty$ NEMA H.A

, NEMA V.A.

$\lim_{x \to \pm\infty} \frac{\sqrt[3]{x^2(x-1)^2}}{x} = \lim_{x \to \pm\infty} \sqrt[3]{\frac{x^2(x-1)^2}{x^3}} = \lim_{x \to \pm\infty} \sqrt[3]{\frac{(x-1)^2}{x}} = \lim_{x \to \pm\infty} \sqrt[3]{x - 2 + \frac{1}{x^2}} = \pm\infty \Rightarrow$ NEMA K.A

$f'(x) = \left[(x^2 - x)^{\frac{2}{3}}\right]' = \frac{2}{3}(x^2 - x)^{-\frac{1}{3}} \cdot (2x - 1) = \frac{2}{3} \cdot \frac{2x - 1}{\sqrt[3]{x^2 - x}}$

$f'(x) = 0 \Rightarrow x_S = \frac{1}{2}$

$f'(x) > 0 \Rightarrow \frac{2}{3} \cdot \frac{2x - 1}{\sqrt[3]{x^2 - x}} > 0 \Rightarrow$

	$-\infty$	0	$1/2$	1	$+\infty$
$2x - 1$	$-$	$-$ 0	$+$	$+$	
$x^2 - x$	$+$ 0	$-$	$-$ 0	$+$	
$f'(x)$	$-$	$+$	$-$	$+$	

$f'(x) > 0 \Rightarrow x \in (0, \frac{1}{2}) \cup (1, +\infty) \quad f \nearrow$

$f'(x) < 0 \Rightarrow x \in (-\infty, 0) \cup (\frac{1}{2}, 1) \quad f \searrow$

$f'(x) = \frac{2}{3} \frac{2x - 1}{(x^2 - x)^{\frac{1}{3}}} \Rightarrow f''(x) = \frac{2}{3} \cdot \frac{2 \cdot (x^2 - x)^{\frac{1}{3}} - (2x - 1) \cdot \frac{1}{3}(x^2 - x)^{-\frac{2}{3}} \cdot (2x - 1)}{(x^2 - x)^{\frac{2}{3}}}$

$f''(x) = \frac{2}{3} \cdot \frac{2(x^2 - x) - (2x - 1)^2}{(x^2 - x)^{4/3}} = \frac{2}{3} \frac{2x^2 - 2x - 4x^2 + 4x - 1}{(x^2 - x)^{4/3}} = \frac{2}{3} \cdot \frac{-2x^2 + 2x - 1}{(x^2 - x)^{4/3}}$

$f''(x) = -\frac{2}{3} \cdot \frac{2x^2 - 2x + 1}{(x^2 - x)^{4/3}}$ $\qquad 2x^2 - 2x + 1 > 0 \quad \forall x$ DER JE $D = 4 - 8 < 0$

$f''(x) < 0, \forall x \in$ D.P. $\quad f$-JA KONKAVNA $\cap$

$f''(\frac{1}{2}) < 0 \Rightarrow$; LOKALNI MAX