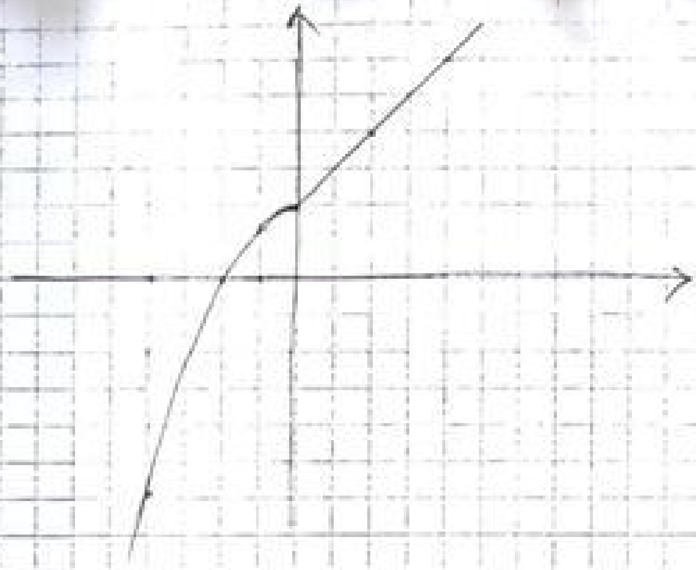


2043.

$$y = \begin{cases} e^{-x} + 1, & x \geq 0 \\ \lambda + x, & x < 0 \end{cases}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(e^{-x} + 1 \right) = 2$$

$$\lim_{x \to 0_+} f(x) = \lim_{x \to 0_+} (\lambda + x) = \lambda$$

$$\lim_{x \to 0_-} f(x) = \lim_{x \to 0_-} \left(e^{-x} + 1 \right) = 2$$

- da bi funk. bila neprekidna u tački 0 potrebno je da je:

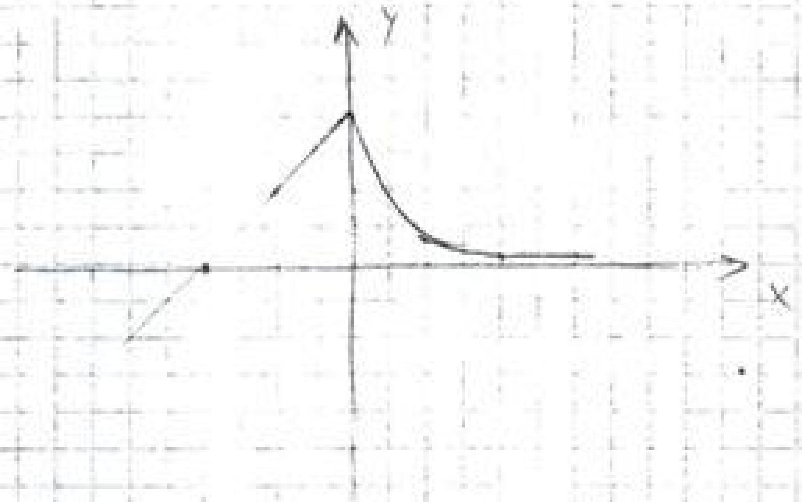
$$\lim_{x \to 0_-} f(x) = \lim_{x \to 0_+} f(x) \quad \text{tj.} \quad \boxed{\lambda = 2}$$

$y = e^{-x} + 1 = \frac{1}{e^x} + 1$

x	0	1	2
y	2	0,367	0,14

$y = x + 2$

x	-2	-1
y	0	1



IZVODI (Bakir)

$f: E \to R$

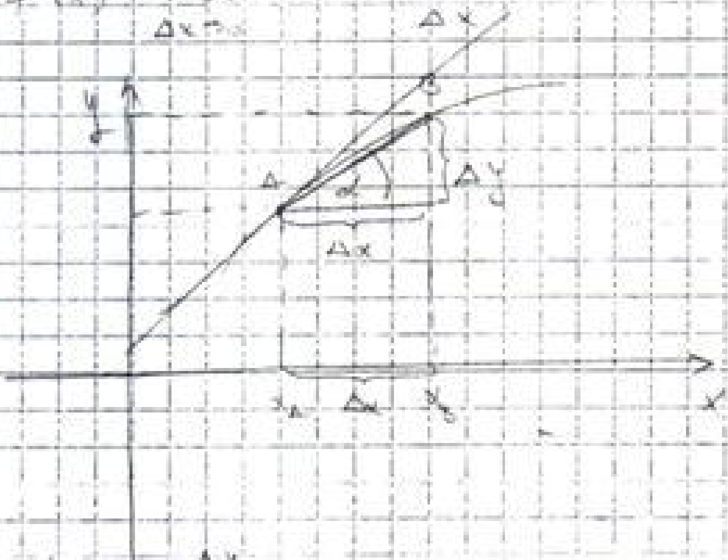
$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$ — za više vrijednosti f

$x = x_0 + \Delta x$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$ — za konkretne tačke

$x \nearrow x_0$ - lijevi izvod
$x \searrow x_0$ - desni — " —
} = postoji izvod f.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



$$\operatorname{tg} \alpha = \frac{\Delta y}{\Delta x}$$

$$\Delta x \to 0 \quad \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = f'(x)$$

* neprekidnost - potreban uslov za postojanje izvoda

- fizikalna interpretacija - preko brzine

$$\frac{\Delta s}{\Delta t} = v_{sr}$$

- u konačnom graničnom procesu kad $\Delta s \to 0$ v_{sr} postaje $v_{tr.}$

$$\big(f(x) + g(x)\big)' = f'(x) + g'(x)$$

$$\big(\alpha \cdot f(x)\big)' = \alpha \cdot f'(x)$$

$$\big(\alpha f(x) + \beta g(x)\big)' = \alpha f'(x) + \beta g'(x)$$ - svojstvo linearnosti

$$\big(f(x) \cdot g(x)\big)' = f'(x)\, g(x) + f(x) \cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\, g(x) - f(x)\, g'(x)}{g^2(x)} \qquad g(x) \neq 0$$

$$(\Delta x \to 0 \land \Delta x \neq 0)$$

(1) P

$y = f(x) = x^2$ za $(\forall x)$

$D(f) = \mathbb{R}^2$, elementarna f., na svom $D(f)$ neprekidna

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2\Delta x x + \Delta x^2 - \cancel{x^2}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x + \Delta x^{\to 0}}{} = 2x$$

$$f'(x) = (x^2)' = 2x \quad (\forall x \in \mathbb{R})$$

$$f'(5) = 2 \cdot 5 = 10$$

$$f'(5) = \lim_{x\to 5} \frac{f(x)-f(5)}{x-5} = \lim_{x\to 5} \frac{x^2-25}{x-5} = \lim_{x\to 5} (x+5) = 10$$

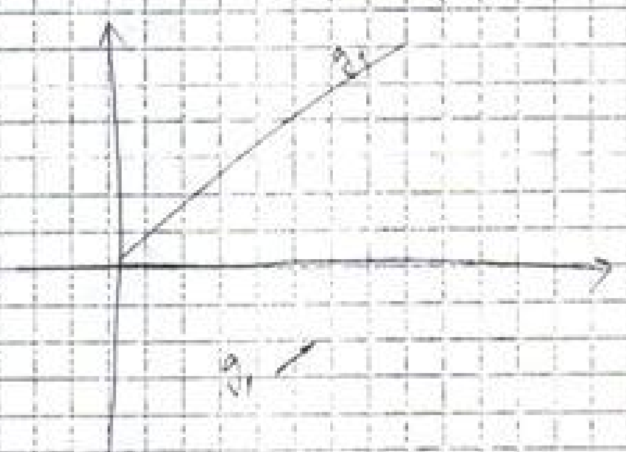
2\. Pokazati da postoji inverzna f. $y = y(x)$ ako je $y + \ln y = x$ te potom naći $y'(x) = y'_x$

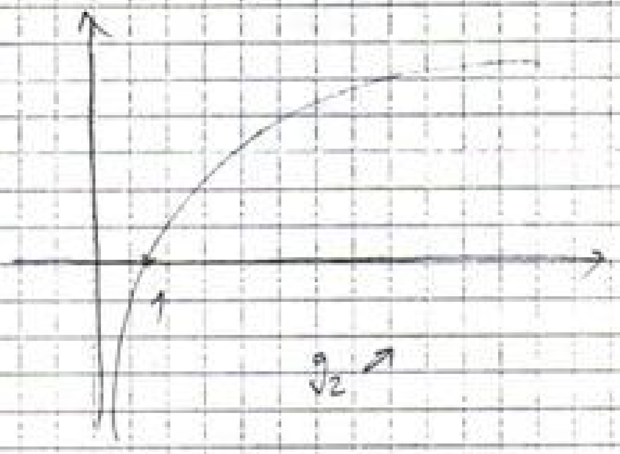
$x = g(y) = y + \ln y$

- treba dokazati inverznu, ako dokažemo da je f injektivna (za različite originale daje različite slike)
- kvadratna f. nije injekcija
- ako dokažemo da je f. monotona onda je bilo koja hor. linija siječe ga u jednom mjestu

• Dokažimo da je $g(y)$ monotona

$$g(y) = \underbrace{g_1(y)}_{y} + \underbrace{g_2(y)}_{\ln y}$$





$x = g(y) = g_1(y) + g_2(y) \nearrow$ monotono rastuća

- sigurno postoji inverzna f. $g^{-1}(x) = y$

$$x'_y = \frac{dx}{dy} = (y + \ln y)' = 1 + \frac{1}{y} = \frac{y+1}{y} = \frac{dx}{dy}$$

$$y'_x = \frac{dy}{dx} = \frac{y}{y+1}$$

$(y > 0)$

(3.) Koristeći pravila traženja neprek. f. odrediti izvod.

$y = x^x$

x^x - spepona $\quad e^x$ - exp.

$(x^\alpha)' = \alpha x^{\alpha - 1}$

$(e^x)' = e^x$

I $\quad x^x = e^{x \ln x}$

$$y' = (e^{x \ln x})' = e^{x \ln x} \cdot (x \ln x)' = e^{x \ln x} \cdot \left(1 \cdot \ln x + x \cdot \frac{1}{x}\right)$$

$$= x^x (1 + \ln x) \qquad (x > 0)$$

$$\boxed{(\ln y)' = \frac{1}{y} \cdot y'}$$

II $\quad y = x^x \qquad (x > 0)$

$\ln y = x \ln x \quad /'$

$(\ln y)' = \ln x + 1$

$\frac{y'}{y} = \ln x + 1$

$y' = y(\ln x + 1)$

$y' = x^x (\ln x + 1)$

4. Za koje vrijednost $\alpha \in \mathbb{R}$ realna f. data sa

$$f(x)=\begin{cases} x^\alpha \sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$$

a) neprekidna u $x=0$

b) ima izvod u $x=0$

c) ima neprekidan izvod u $x=0$

$(\alpha>0) \wedge (x>0)$

a) $\lim\limits_{x\to 0_+} f(x) = \lim\limits_{x\to 0_+}\left(x^\alpha \sin\frac{1}{x}\right)$

$\lim\limits_{x\to 0_+} \sin\frac{1}{x}$ = ne postoji (jer sin oscilira $+1, -1$ ne postiže stacionarni vrijednost)

$\lim\limits_{x\to 0} x^\alpha = 0$ ($x>0$)

$\lim \underbrace{x^\alpha}_{0} \underbrace{\sin\frac{1}{x}}_{\in[-1,1]\ \text{ograničena}} = 0 \ (\alpha>0) = f(0)$

- za $\alpha>0$ f. je nep.

b) $f'(0) = \lim_{x \searrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \searrow 0} \frac{x^{\alpha} \sin\frac{1}{x} - 0}{x} =$

$= \lim_{x \searrow 0} x^{\alpha-1} \sin\frac{1}{x}$ - ako ovo postoj postoji izvod

za $\alpha > 1$ postoji ovaj lim $\Rightarrow$ postoji izvod

$f'(0) = 0$

c) $f'(x)$

$\lim_{x \searrow 0} f'(x) = f'(0)$ $\qquad \left(\frac{1}{x}\right)' = \frac{-1}{x^2}$

$\lim_{x \searrow 0} f'(x) = \lim_{\substack{x \searrow 0 \\ x \neq 0}} \left(\alpha x^{\alpha-1} \sin\frac{1}{x} + x^{\alpha} \cos\frac{1}{x} \cdot \frac{-1}{x^2}\right)$

$\lim_{x \searrow 0} f'(x) = \lim_{x \searrow 0} \left(\underbrace{\alpha x^{\alpha-1} \sin\frac{1}{x}}_{\alpha - 1 > 0} - \underbrace{x^{\alpha-2} \cdot \cos\frac{1}{x}}_{\alpha - 2 > 0}\right)$

$(\alpha > 1) \wedge (\alpha > 2)$

$\boxed{\alpha > 2}$

f. ima neprekidan izvod

5. Dokazati uniformnost funkcije

$f(x) = \ln x$

$x \in (0, +\infty)$

1° postoji vrijednost f. za $x = x_0$

$f(x_0)$

$f(0) = \ln 0 = \log_e 0 \quad f(0) = 1$

2° Postoji lijeva i desna gr. vrijednost

za $x = x_0$

$f(x_0 - 0) \neq f(x_0 + 0)$

$$\left.\begin{array}{l} \lim_{x \to 0_-} f(x) = \lim_{x \to 0_-} \ln 0 - 0 = \ln 0 = \log_e 0 \\ \lim_{x \to 0_+} f(x) - \text{uvo dat} \end{array}\right\} \text{nepostoje}$$

3° $f: E \to R$

$(\forall \varepsilon > 0)\ (\exists \delta(\varepsilon) > 0)\ (\forall x_1, x_2 \in E)$

$|x_2 - x_1| < \delta \quad (1)$

iz $(1) \Rightarrow |f(x_2) - f(x_1)| < \varepsilon$

$$|x_2 - x_1| = \left| \frac{1}{e^n} - \frac{1}{e^{n-1}} \right| = \left| \frac{e-1}{e^n} \right| = \frac{e-1}{e^n}$$

$(n > 0)$

kad

$$\left| \ln \frac{1}{e^{n-1}} - \ln \frac{1}{e^n} \right| = \left| \ln \frac{\frac{1}{e^{n-1}}}{\frac{1}{e^n}} \right|$$

$$= \left| \ln \frac{e^n}{e^{n-1}} \right| = \ln \frac{e^n}{e^n \cdot e^{-1}} = |\ln e| = 1 - \text{ov.!}$$

~~ln x~~ = ne postoji uniforma neprek.

FUNKCIJE

$f: X \xrightarrow{\text{pravila}} Y \to$ područje def.

↓

oblast def.

- PARNOST

$\boxed{f(-x) = f(x)}$ parna (simetričan u odnosu na y-om) → (nema injekcije)

$\boxed{f(-x) = -f(x)}$ neparna (– " – – " – na $(0,0)$)

- PERIODIČNOST

i) $\forall (x \in D) \quad x + p \in D$

ii) $\forall (x \in D) \quad f(x+p) = f(x)$ — p-osnovni period f.

- MONOTONOST $(\forall x_1, x_2 \in E)$

1° nespadajuća $x_1 \leq x_2 \Rightarrow f(x_1) \leq f(x_2)$

2° nerastuća $x_1 \leq x_2 \Rightarrow f(x_1) \geq f(x_2)$

3° opadajuća $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

4° rastuća $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

(3°–4°: STROGO; 1°–4°: MONOTONE)

- INVERZNA F.

– MONOTONA I NEPREKIDNA

- OGRANIČENOST

$f: D \to \mathbb{R}$, $(p, P \in \mathbb{R})$ $(E \subseteq D)$ $(f(x) \mid x \in E)$

1° $f(x) < P$ – odozgo

2° $f(x) > p$ – odozdo

(OGRANIČENA na E)

• LIMES FUNKCIJE

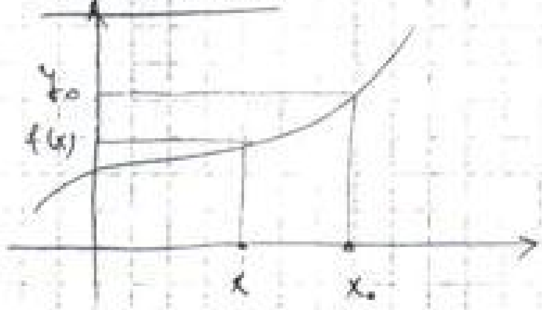
$f: X \to Y$

x_0 - tačka gomilanja X

$y_0 \in \mathbb{R}$ - limes f u x_0

$f \to y_0$ kada $x \to x_0$

KONAČAN



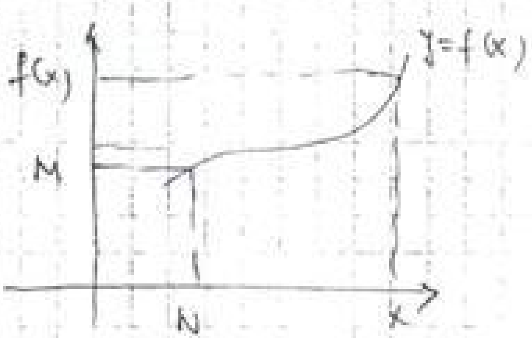
$\lim\limits_{x \to x_0} f(x) = y_0$

$y_0 \in \mathbb{R}$ - KONAČAN LIM

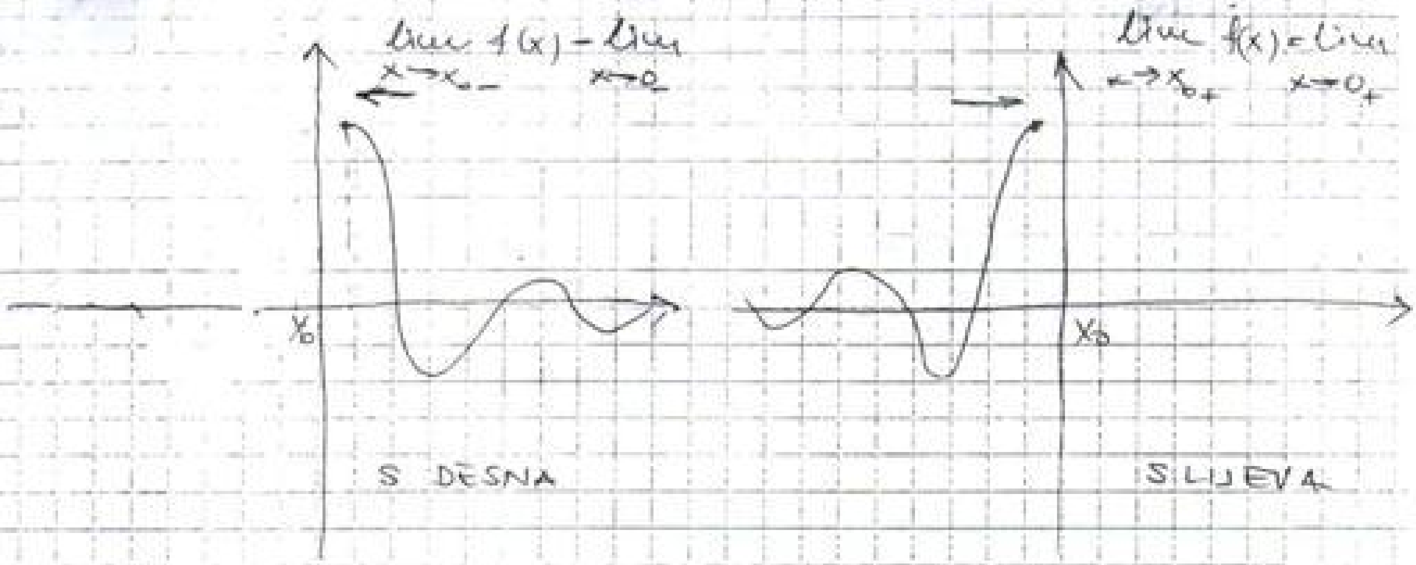
$y_0 = \pm\infty$ - BESKONAČNA LIM

$$\left(\lim_{x \to x_0} f(x) = y_0\right) \Leftrightarrow (\forall \varepsilon > 0)(\exists \delta := \delta(\varepsilon) > 0)$$
$$\left(|x - x_0| < \delta \Longrightarrow |f(x) - y_0| < \varepsilon\right)$$

BESKONAČAN



$$\left(\lim_{x \to +\infty} f(x) = +\infty\right) \Longleftrightarrow (\forall M \in \mathbb{R})(\exists N \in \mathbb{R})\ (\forall x \in X)$$
$$(x > N \Longrightarrow f(x) > M)$$



- ako je $\lim_{x \to x_{0+}} f(x) \neq \lim_{x \to x_{0-}} f(x) \Rightarrow \lim_{x \to x_0} f(x)$ $\boxed{\text{ne postoji}}$

$$\lim_{x \to x_{0+}} f(x) = \lim_{x \to x_{0-}} f(x) = \lim_{x \to x_0} f(x)$$

- Da li $\lim_{x \to x_0} f(x) = y$ postojao mora (niz x_n)

$$\lim_{n \to \infty} x_n = x_0 \Rightarrow \lim_{n \to \infty} f(x_n) = y_0$$

OSOBINE

1) $\lim_{x \to x_0} f(x) = y_0 \Rightarrow \lim_{x \to x_0} |f(x)| = |y_0|$

2) $\lim_{x \to x_0} f(x) < \lim_{x \to x_0} g(x) \Rightarrow f(x) < g(x)$

$\lim_{x \to x_0} f(x) < c \Rightarrow f(x) < c$

3) DVA ŽANDARA

$f(x) \leq g(x) \leq h(x)$

$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} h(x) = b \Rightarrow \lim_{x \to x_0} g(x) = b$

• PREKIDI

NEPREKIDNOST

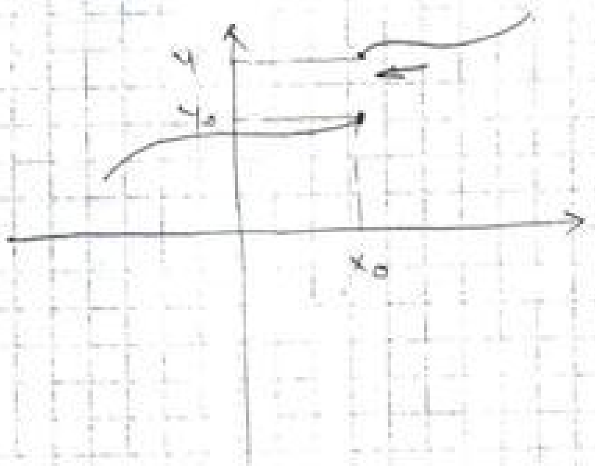
$f: D \to K$

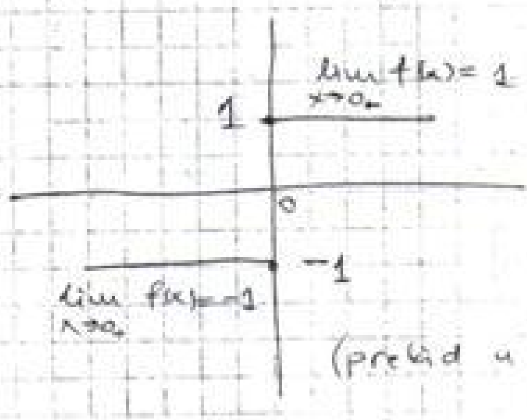
1. neprekidna u $(x_0 \in D)\ (\forall \varepsilon > 0)\ (\exists \delta = \delta(\varepsilon) > 0)$

$$|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$$

PREKIDNOST

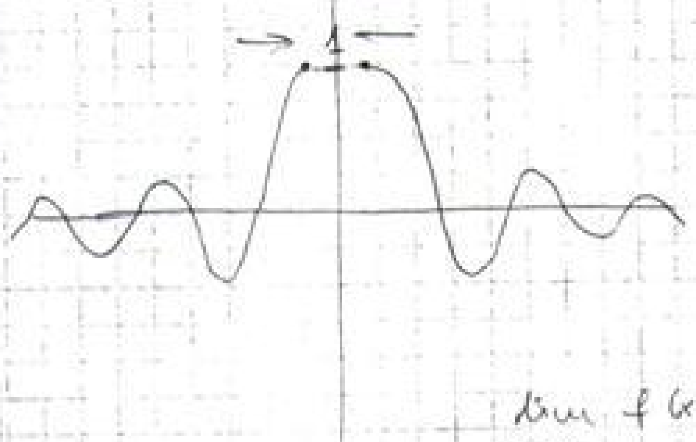
1. vrste $\lim\limits_{x \to x_{0+}} f(x) \neq \lim\limits_{x \to x_{0-}} f(x)$ (neotklonjiv)





(prekid u 0)

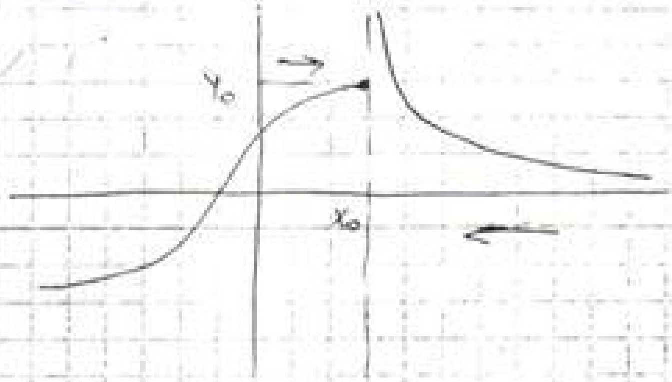
(otklonjiv)



npr: $\frac{\sin x}{x}$

$$\lim_{x \to 0_-} f(x) = \lim_{x \to 0_+} f(x) = 1$$

(2. vrste) - jedan od $\lim (f(x)) = \pm\infty$



$\lim_{x \to x_{0-}} f(x) = y_0$

$\lim_{x \to x_{0+}} f(x) = \pm\infty$ — asimptota

ASIMPTOTE

K.A. $y = kx + n$

$k = \lim_{x \to \pm\infty} \frac{f(x)}{x}$ $n = \lim_{x \to \pm\infty} [f(x) - kx]$

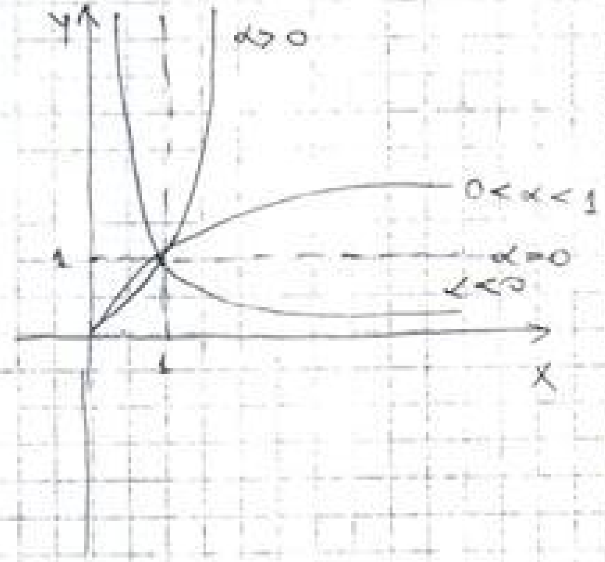
V.A. $\lim_{x \to x_0 \pm 0} f(x) = \pm\infty$ $x = x_0$ — V.A.

H.A. $y = n$

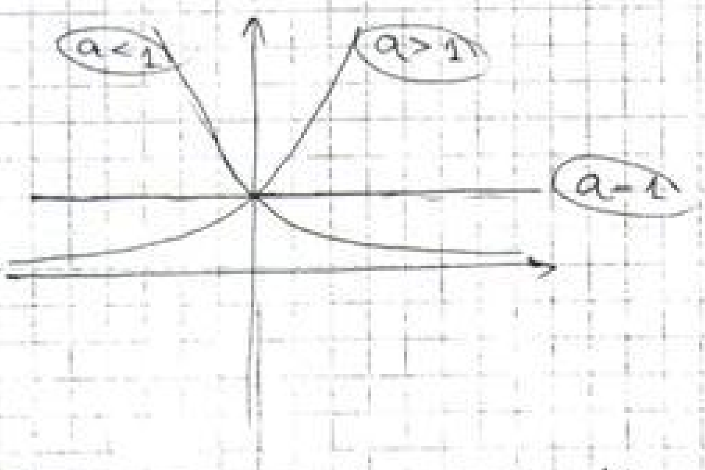
$\lim_{x \to \pm\infty} f(x) = n$

• ELEMENTARNE FUN

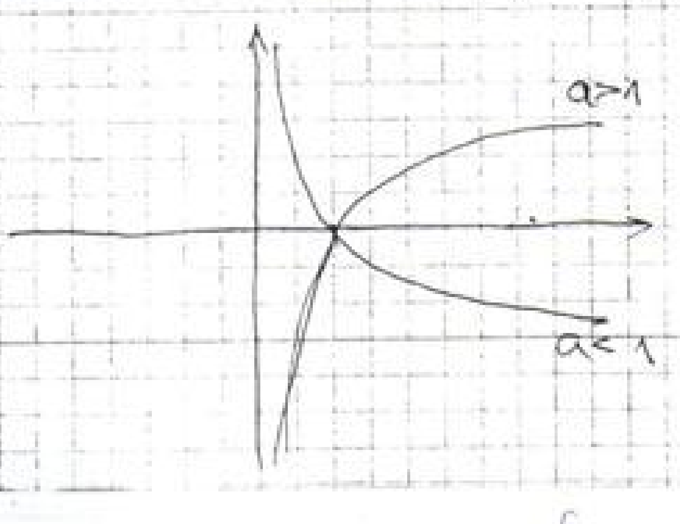
STEPENA $y = x^{\alpha}$, $x > 0$



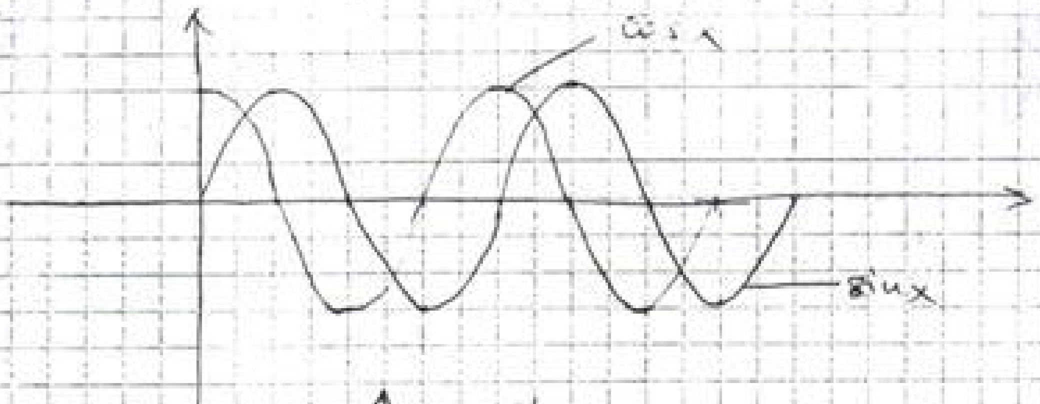
EKSPONENCIJALNA $y = a^{x}$

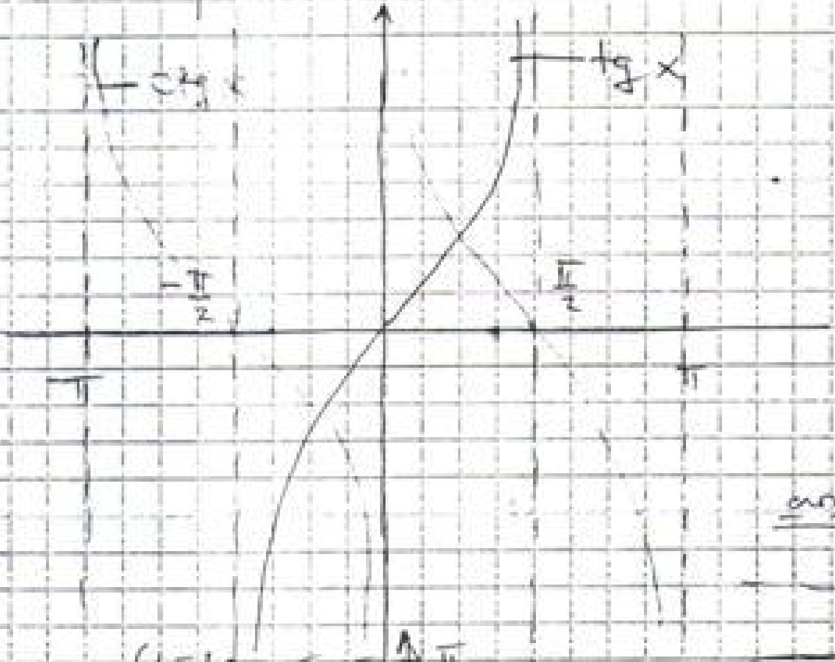


LOGARITAMSKA $y = \log_a x$, $a \neq 1$, $x > 0$

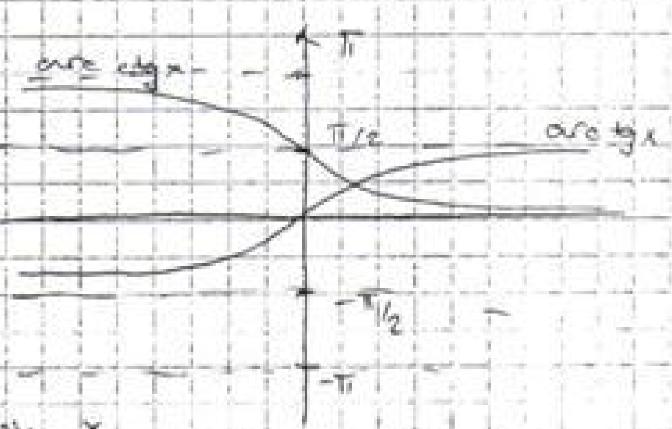


TRIGONOMETRIJSKE

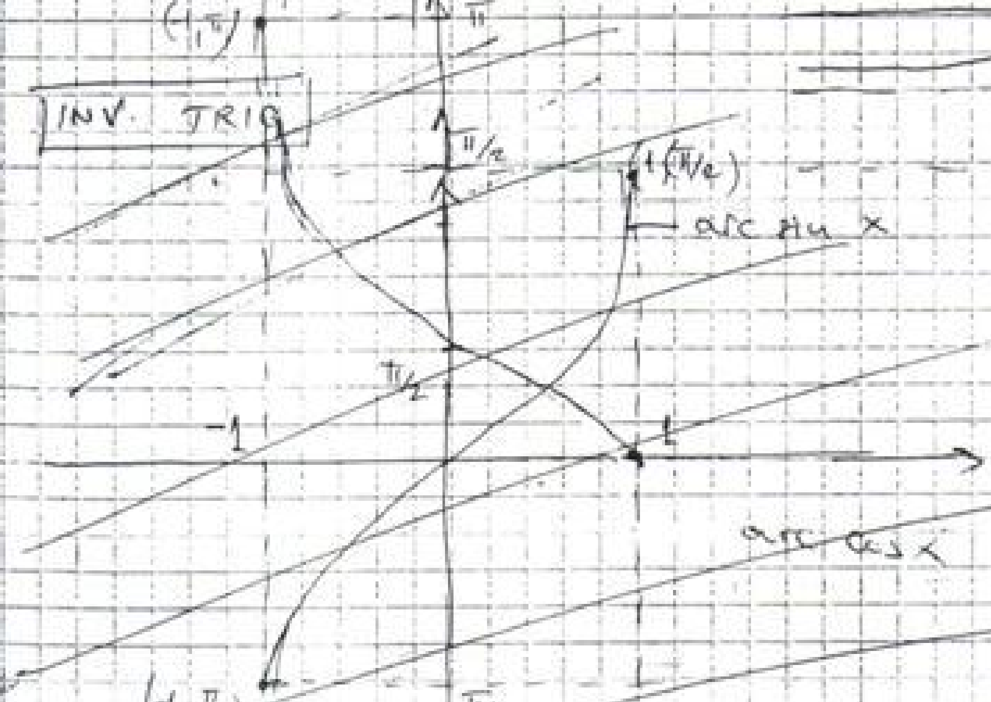




INV. TRIG arc tg i arc ctg



INV. TRIG



SKICIRANJE FUNKCIJA

I $\Rightarrow$ $y = f(x)$

II $y = A \cdot f(x)$

$|A| > 1$ — graf je — viši

$|A| < 1$ — graf je — niži

III $y = f(bx)$

$|b| > 1$ $\Rightarrow$ sabijanje

$|b| < 1$ $\Rightarrow$ razvlačenje

IV $y = f(x + c)$

$+c$ — pomak po x-osi — $\leftarrow$ L

$-c$ — pomak po x-osi — $\rightarrow$ D

V $y = f(x) \pm b$

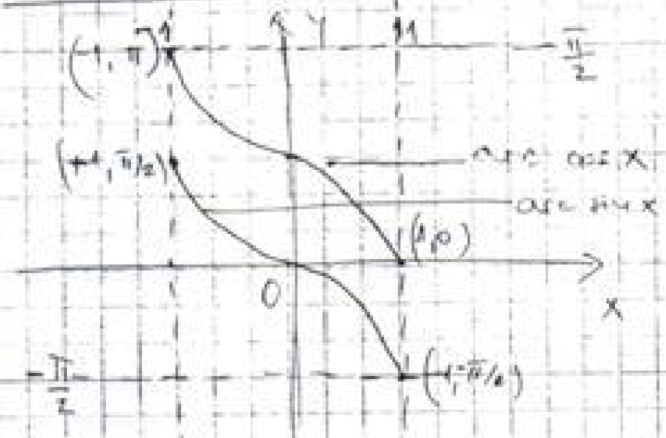
$+b$ — po y-osi — $\uparrow y$

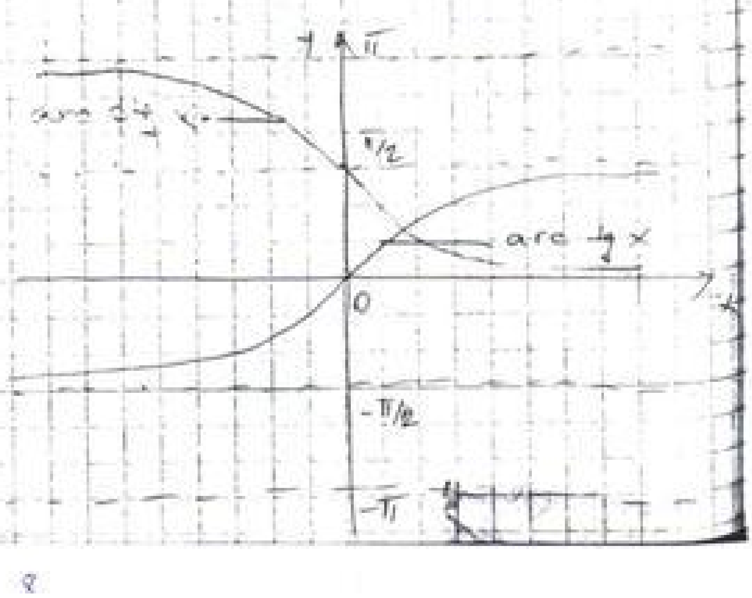
$-b$ — po y-osi — $\downarrow y$

VI $y = -f(x)$ — simetrično sa — x-osom

$y = f(-x)$ — simetrično sa — y-osom

INVERZNA TRIG

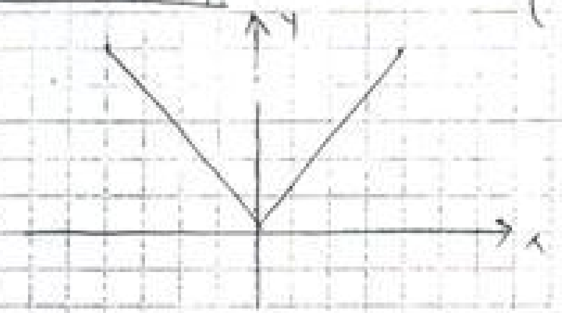




SPECIJALNE FUN

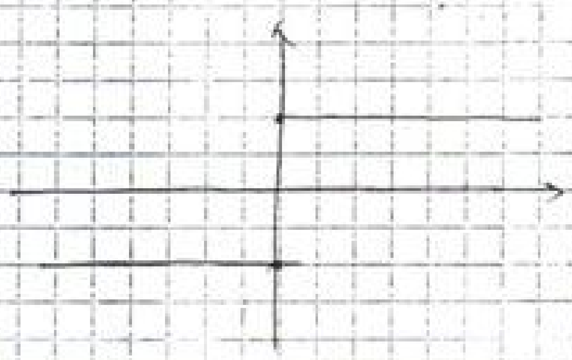
$\boxed{y = |x|}$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



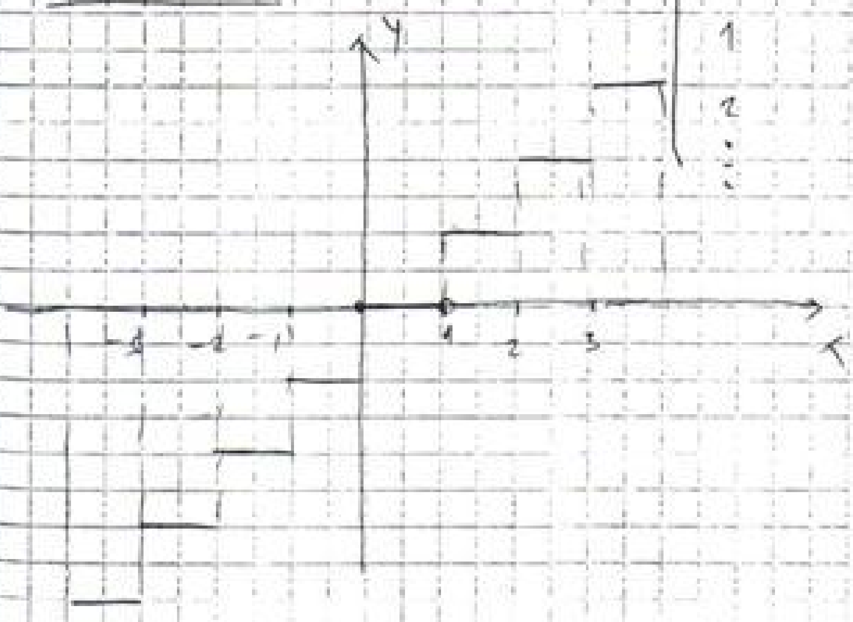
$\boxed{y = \operatorname{sgn} x}$

$$\operatorname{sgn} x = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$



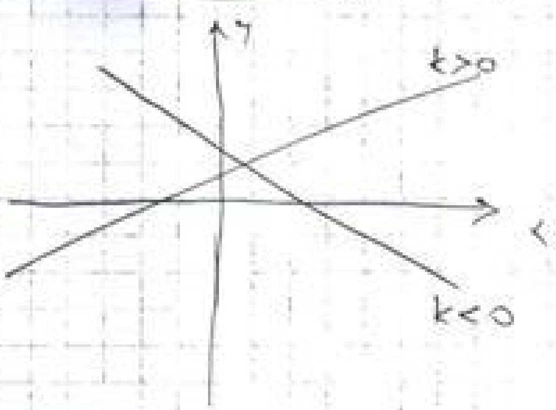
$\boxed{y = \lfloor x \rfloor}$

$$\lfloor x \rfloor = \begin{cases} \vdots \\ -2 & x \in [-2, -1) \\ -1 & x \in [-1, 0) \\ 0 & x \in [0, 1) \\ 1 & x \in [1, 2) \\ 2 & x \in [2, 3) \\ \vdots \end{cases}$$

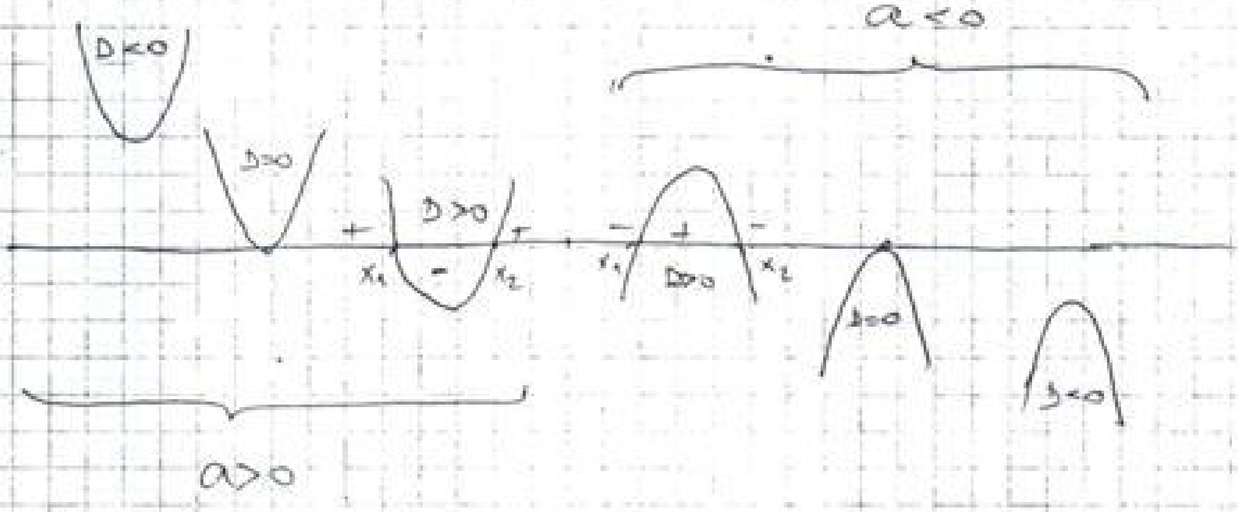


LINEARNA F.

$y = kx + n$

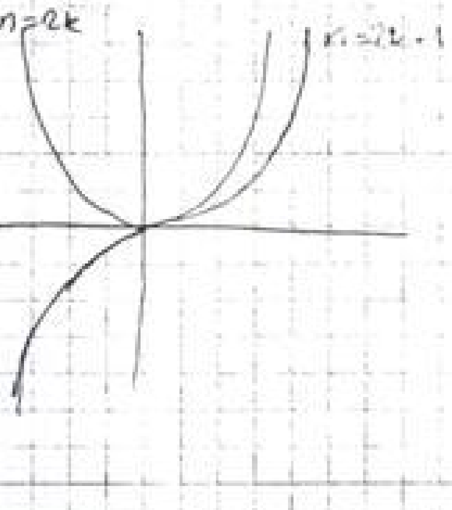


KVADRATNA F.

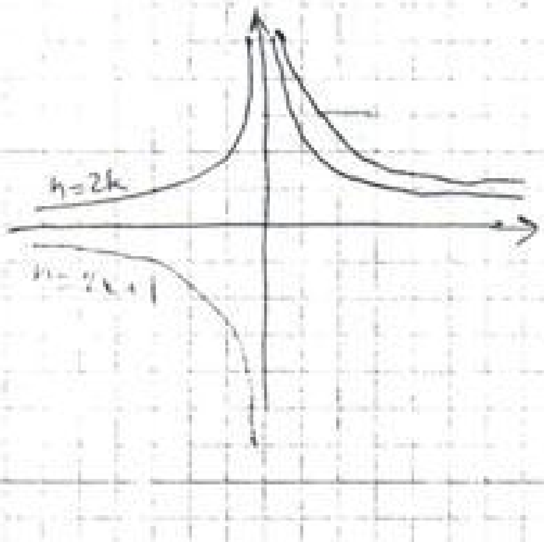


STEPENA F.

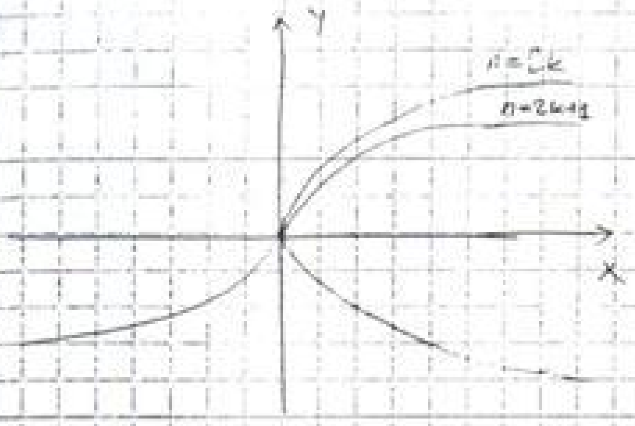
$\boxed{y = x^n}$



$\boxed{y = \frac{1}{x^n}}$



$$y = \sqrt[n]{x} = x^{\frac{1}{n}} \quad \left(0 < \frac{1}{n} < 1\right)$$



TUTORIJA 6.

Fatkić - Mesihović

108.) $D(f) = ?$

a)

$$f(x) = 2 + \sqrt{3} - \frac{(4-x^2)^{\frac{1}{2}}}{1+x}$$

$\sqrt{4-x^2} \geq 0$ $\qquad\qquad$ $1+x \neq 0$

$4 - x^2 \geq 0$ $\qquad\qquad$ $x \neq -1$

$x^2 \leq 4$

$|x| \leq \pm 2$ $\qquad\qquad$ $D(f) = [-2, -1) \cup (-1, 2]$

$-2 \leq x \leq 2$

b)

$$f(x) = \begin{cases} x\sqrt{x^2-2} & \text{za} \quad x < 10 \\ \sqrt{x+1} & \text{za} \quad x \in [10, 20] \\ \sqrt{x^2-2} & \text{za} \quad x \in [20, +\infty) \end{cases}$$

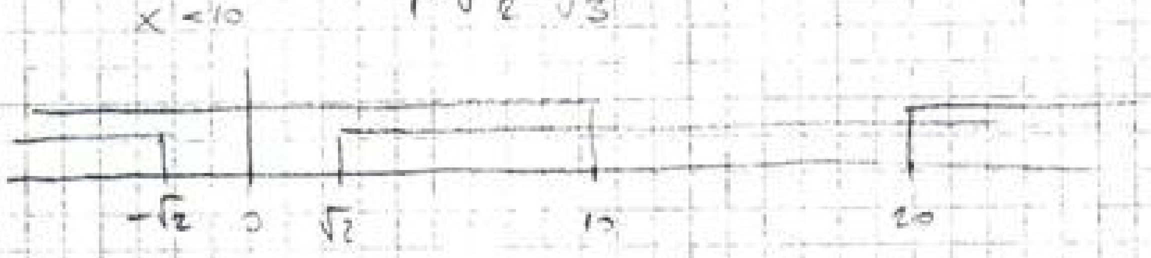
1° $x \neq 0$ $\qquad$ 2° $x+1 \geq 0$ $\qquad$ 3° $x^2 - 2 \geq 0$

$x^2 - 2 \geq 0$ $\qquad$ $x \geq -1$ $\qquad$ $|x| \geq \pm\sqrt{2}$

$x^2 \geq 2$ $\qquad$ $x \in [10, 20]$ $\qquad$ $x \in [20, +\infty)$

$|x| \geq \pm\sqrt{2}$

$x < 10$ $\qquad$ $1° \cup 2° \cup 3°$



$D(f) = (-\infty, \sqrt{2}] \cup [\sqrt{2}, 20] \cup [10, +\infty)$

d) $f(x) = (1-x-\sqrt{2})^{\frac{9}{2}} + \sqrt{\frac{x^2-x+2}{|x-1|+1}}$

1°

$1-x-\sqrt{2} \geq 0$

$-x \geq \sqrt{2}-1 \quad /(-1)$

$x \leq 1-\sqrt{2}$

2°

$\frac{x^2-x+2}{|x-1|+1} \geq 0$

$|x-1|+1>0$ za $\forall x \in R$

$x^2-x+2=0$

$x_{1,2} = \frac{1 \pm \sqrt{1-8}}{2}$ $\Rightarrow$ $D<0$

$(+) \; x^2-x+2>0$ za $\forall x \in R$

$D(f) = (-\infty, 1-\sqrt{2}]$

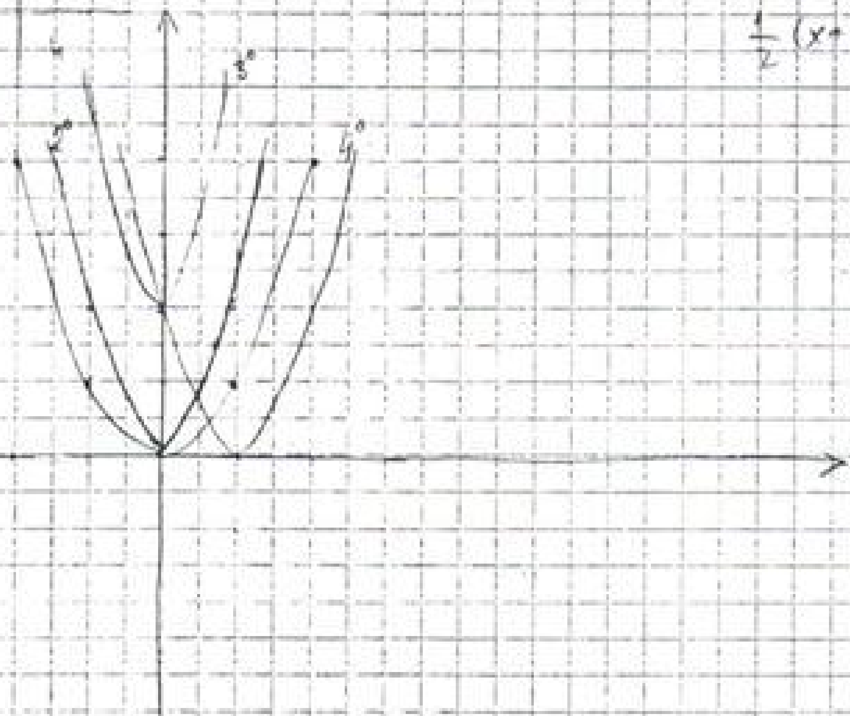
(109.)

g)

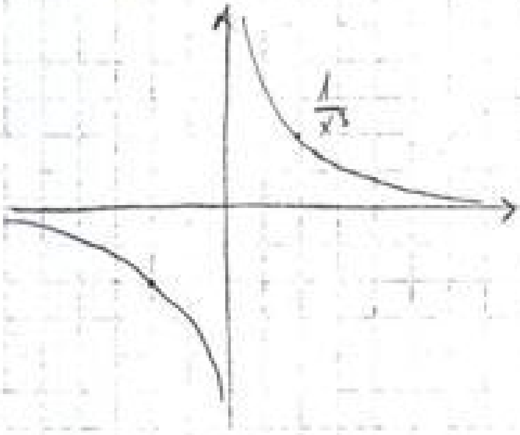
$f(x) = 2x^2$ (1°), $2(x+1)^2$ (2°), $2x^2+2$ (3°), $2(x-1)^2+2$ (4°), $\frac{1}{2}x^2+x+1$

$\frac{1}{2}(x+1)^2+\frac{1}{2}$

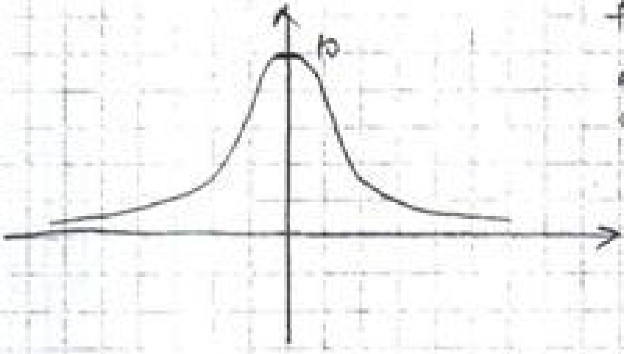
x	-2	-1	0	1	2
x^2	4	1	0	1	4



g) $f(x) = \frac{1}{x^3}$ (3 = 2k+1)



$f(x) = \frac{10}{x^2+1}$



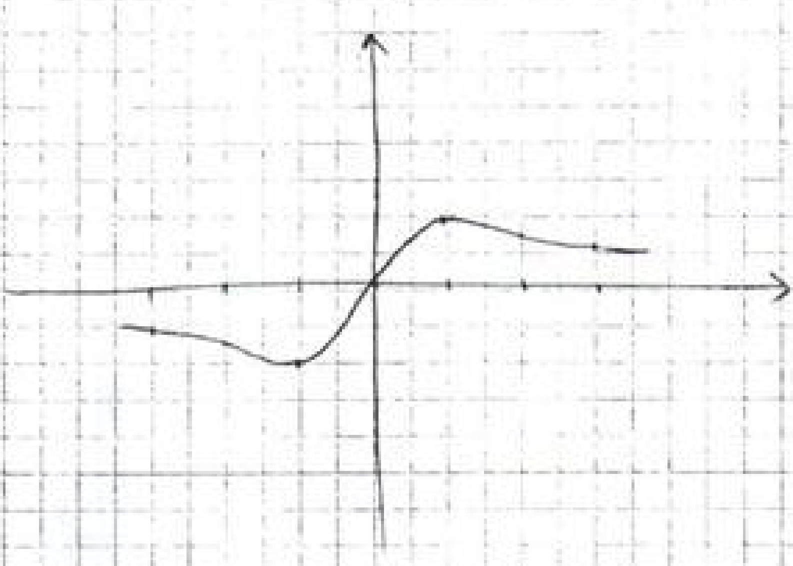
$f(-x) = f(x)$ – simetr. ćna u odnosu na y

min za $x=0$

max za $x=0$

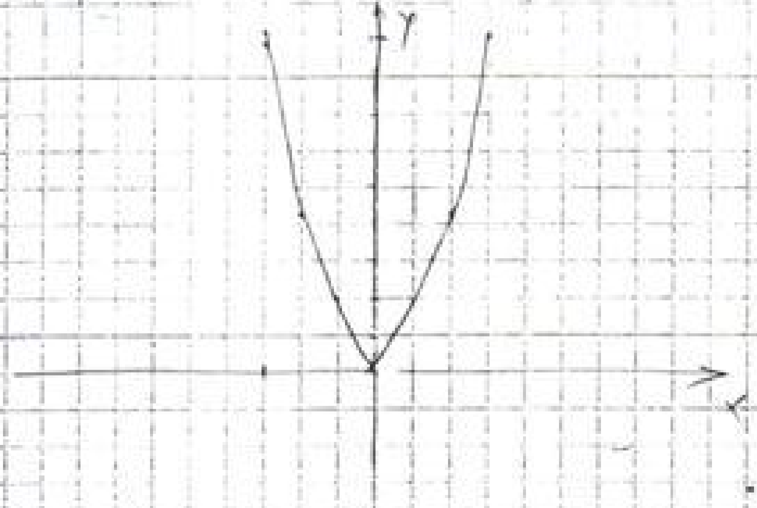
$f(x) = \frac{2x}{x^2+1}$

x	-3	-2	-1	0	1	2	3
f(x)	-0,6	-0,8	-1	0	1	0,8	0,6



$f(x) = x^2 + \frac{1}{x^2}$

x	-3	-2	-1	0	1	2	3
y	9,1	4,25	2	0	2	4,25	9,1



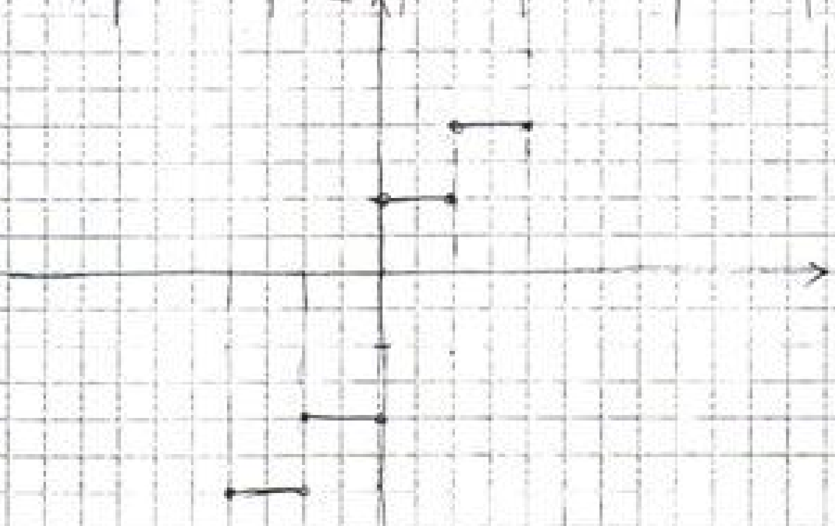
(107.)

a) $f(x) = sgn(x) + [x]$

$$sgn\, x = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

[x] cap. cu dis intervala

x	[-2,-1)	[-1,0)	0	(0,1]	(1,2]	...
f(x)	-3	-2	0	1	2	...

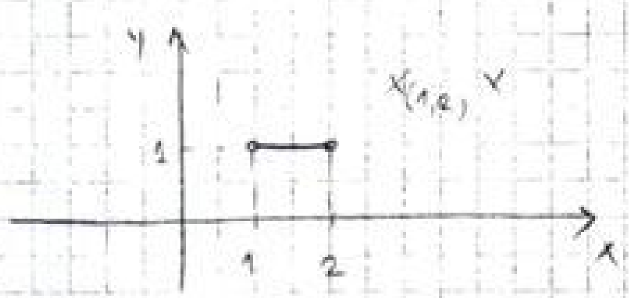


b) $\chi_A(x)$, $\chi_A(x)\cdot\chi_B(x)$

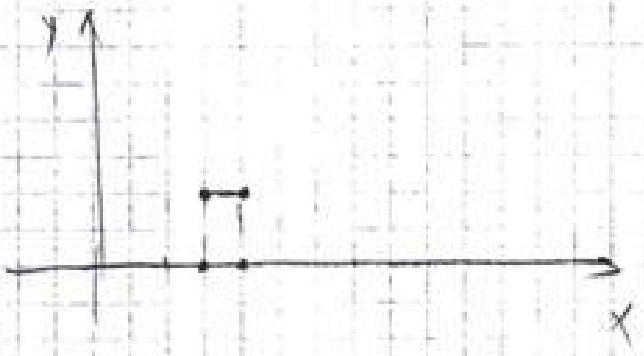
za $A=(1,2)$, $B=\left(\frac{3}{2},4\right]$

$$\chi_A(x)=\begin{cases}1 & x\in A\\ & \text{za}\\ 0 & x\notin A\end{cases}$$

x	$(-\infty,1]$	$(1,2)$	$[2,+\infty)$
$\chi_A(x)$	0	1	0



x	$\chi_{(1,2)}(x)\cdot\chi_{(\frac{3}{2},4]}(x)$	
$(-\infty,1]$	$0\cdot 0$	0
$\left(1,\frac{3}{2}\right)$	$1\cdot 0$	0
$\left(\frac{3}{2},2\right)$	$1\cdot 1$	1
$[2,4]$	$0\cdot 1$	0
$(4,+\infty)$	$0\cdot 0$	0



114.

e) $f(x) = \frac{\operatorname{sgn}(x)}{1+|x|}$

- konveksnost

$$f\left(\frac{x_1+x_2}{2}\right) \leq \frac{f(x_1)+f(x_2)}{2}$$

za $x > 0$

$$f\left(\frac{x_1+x_2}{2}\right) \leq \frac{\frac{1}{x_1+1} + \frac{1}{x_2+1}}{2} \; ?$$

$$\frac{1}{x_1+x_2+1}$$

115.

b) $f(x) = \sin \frac{3x-2}{5} + \cos \frac{\pi}{2} x$

$f_1(x) = \sin \frac{3x-2}{5}$

$f_1(x+T) = \sin \frac{3(x+T)-2}{5}$

$= \sin \frac{3x+3T-2}{5}$

$= \sin \frac{3x-2}{5}$ za $\frac{3T}{5} = 2\pi$

$3T = 10\pi$

$T_1 = \frac{10\pi}{3}$

$f_2(x) = \cos \frac{\pi}{2} x = \cos \frac{\pi}{2}(x+T)$

$= \cos\left(\frac{\pi}{2} x + \frac{\pi}{2} T\right)$

$\frac{\pi}{2} T = 2\pi$

$\frac{T}{2} = 4$

$\frac{T_1}{T_2} = \frac{10\pi}{12} \rightarrow$ nije period

nije periodična fn

d) $f(x) = 3x \cos x$

$f(x+T) = 3(x+T)\cos(x+T) = (3x + 3T)(\cos x + \cos T)$

$T = 0$ $\quad = \cancel{3x\cos x} +$

$T = 2\pi$ $\quad 3(x+T) = 3x$ ssi $3T = 0 \Rightarrow T_1 = 0$

$\cos(x+T) = \cos x$ ssi $\cos T = 2\pi \Rightarrow T_2 = 2\pi$

Saudi

$f(x) = \cos x + \sin 2x$

$f_1(x) = \cos x$

$f_1(x+T) = \cos(x+T) = \cos x$ ssi $T_1 = 2\pi$

$f_2(x) = \sin 2x$

$f_2(x+T) = \sin 2(x+T) = \sin(2x + 2T) = \sin 2x$ ssi $2T = 2\pi$

$T_2 = \pi$

$T = 2\pi$

i) $f(x) = 3\cos(4x+6) + \sin x \cos x + \cos^4 x + \sin^4 x$

$f_1(x) = 3\cos(4x+6)$

$f_1(x+T) = 3\cos(4(x+T)+6) = 3\cos(4x + 4T + 6)$

$= 3\cos(4x+6)$ ssi $4T = 2\pi \Rightarrow T_1 = \frac{\pi}{2}$

$f_2(x) = \sin x \cos x = \frac{1}{2}\sin 2x$

$f_2(x+T) = \frac{1}{2}\sin 2(x+T) = \frac{1}{2}\sin(2x+2T) = \frac{1}{2}\sin 2x$ ssi

$2T = 2\pi$

$T_2 = \pi$

$$f_3(x_1) = \cos^4 x + \sin^4 x = (\cos^2 x)^2 + (\sin^2 x)^2$$

$$= \left(\frac{1+\cos 2x}{2}\right)^2 + \left(\frac{1-\cos 2x}{2}\right)^2 =$$

$$= \frac{1}{2}(1+\cos^2 2x) = \frac{1}{2} + \frac{1}{2}\cdot\frac{1+\cos 4x}{2}$$

$$= \frac{3}{4} + \frac{1}{4}\cos 4x \Rightarrow T_3 = \frac{\pi}{2}$$

$$T = \pi$$

2) $f(x) = \cos^6 x + \sin^6 x + 2 =$

$$= (\cos^2 x)^3 + (\sin^2 x)^3 + 2$$

$$= \underbrace{(\cos^2 x + \sin^2 x)}_{1}(\cos^4 x - \cos^2 x \cdot \sin^2 x + \sin^4 x) + 2 =$$

$$= \cos^4 x + 2\cos^2 x \sin^2 x + \sin^4 x - 3\sin^2 x \cos^2 x + 2 =$$

$$= \underbrace{(\cos^2 x + \sin^2 x)}_{1} - 3\sin^2 x \cos^2 x + 2$$

$$= 1 - 3\cos^2 x \sin^2 x + 2 = 1 - 3\left(\frac{1}{2}\sin 2x\right)^2 + 2 =$$

$$= 3 - 3\cdot\frac{1}{4}\cdot\frac{1-\cos 4x}{2}$$

$$= 3 - 3\,\frac{1-\cos 4x}{8} = 3 - \frac{3}{8} + \frac{3\cos 4x}{8}$$

$$f(x+T) = 3 - \frac{3}{8} + \frac{3}{8}\cos 4(x+T)$$

$$4T = 2\pi \Rightarrow T = \frac{\pi}{2}$$

2)

$$f_3(x) = \cos^4 x + \sin^4 x = \cancel{(\cos^2 x)^2 + (\sin^2 x)^2} =$$

$$\cancel{= \left(\frac{1+\cos 2x}{2}\right)^2 + \left(\frac{1-\cos 2x}{2}\right)^2 =}$$

$$\cancel{= \frac{1}{2}\left(1 + \cos^2 2x\right)}$$

$$= \cos^4 x + 2\cos^2 x \sin^2 x + \sin^4 x - 2\cos^2 x \sin^2 x$$

$$= \underbrace{\left(\cos^2 x + \sin^2 x\right)^2}_{1} - 2\cos^2 x \sin^2 x$$

$$= 1 - 2\left(\frac{1}{2}\sin 2x\right)^2 = 1 - \frac{1}{2}\,\frac{1-\cos 4x}{2}$$

$$= 1 - \frac{1-\cos 4x}{4} = 1 - \frac{1}{4} + \frac{\cos 4x}{4}$$

$$= \frac{3}{4} + \frac{\cos 4x}{4}$$

$$T = \frac{\pi}{4}$$

37

$$y = 1 - 2^{-2x}$$

$$2^{-2x} = y - 1$$

$$-2x \log 2 = \log(y-1)$$

$$x = \frac{\log(y-1)}{\log\left(\frac{1}{4}\right)}$$

(219.)

$$\lim_{x\to\pm\infty} x\left(\frac{\pi}{4} - \operatorname{arc\,tg}\frac{x}{x+1}\right) = \frac{1}{2}$$

$$y = \frac{\pi}{4} - \operatorname{arc\,tg}\frac{x}{x+1}$$

$$\operatorname{arc\,tg}\frac{x}{x+1} = \frac{\pi}{4} - y$$

$$\frac{x}{x+1} = \operatorname{tg}\left(\frac{\pi}{4} - y\right)$$

$$x = \operatorname{tg}\left(\frac{\pi}{4} - y\right)(x+1)$$

$$x = \operatorname{tg}\left(\frac{\pi}{4} - y\right)x + \operatorname{tg}\left(\frac{\pi}{4} - y\right)$$

$$\operatorname{tg}\left(\frac{\pi}{4} - y\right) = x\left(1 - \operatorname{tg}\left(\frac{\pi}{4} - y\right)\right)$$

$$x = \frac{\operatorname{tg}\left(\frac{\pi}{4} - y\right)}{1 - \operatorname{tg}\left(\frac{\pi}{4} - y\right)} = \frac{1 - \operatorname{tg} y}{2\operatorname{tg} y}$$

$$A = \lim \frac{1 - \operatorname{tg} y}{2\operatorname{tg} y} = \frac{1}{2}$$

$$A \sim \frac{1}{2} \cdot \frac{1}{x}$$

(220.)

a) $$\lim_{x\to -2} \frac{\arcsin(x+2)}{x^2+2x}$$ $x(x+2) = y(y-2)$

Înlocuim $x+2 = y$, $x \to -2$, $y \to 0$

$$\lim_{x\to -2} \frac{\arcsin y}{y(y-2)} = \left| \begin{array}{l} \arcsin y = t \\ y = \sin t \\ y \to 0 \Rightarrow t \to 0 \end{array} \right| = \lim_{t\to 0} \frac{t}{\sin t\,(\sin t - 2)} =$$

$$= \lim_{t \to 0} \frac{1}{\frac{\sin t^{2}}{t}(\sin t - 2)} = \frac{-1}{2}$$

b) ?

221.

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \lim_{x \to 0} \frac{2 \sin^2 \frac{x}{2}}{x^2} =$$

$$= \frac{1}{2} \lim_{x \to 0} \left(\frac{\sin \frac{x}{2}}{\sin \frac{x}{2}} \right)^2 = \frac{1}{2} \lim_{\frac{x}{2} \to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} = \frac{1}{2} \cdot 1^2 = \frac{1}{2}$$

226. ?

(147.)

b) $x = \frac{t}{1-t} \Rightarrow x(1-t) = t$

$x - xt = t$

$y = \cos t$

$x = t + xt \Rightarrow x = t(x+1)$

$y = \cos \frac{x}{x+1}$ $\quad t = \frac{x}{x+1}$

c) $x = |t+1| - 2$

$y = t^2 - 3$ (1)

1° $x = -t - 1 - 2 = -t - 3$ $\Rightarrow$ $t = -x - 3$

2° $x = t + 1 - 2 = t - 1$ $\qquad t = x + 1$

1° $y = (-x-3)^2 - 3 = x^2 + 6x + 9 - 3 = x^2 + 6x + 6$

2° $y = (x+1)^2 - 3 = x^2 + 2x + 1 - 3 = x^2 + 2x - 2$

(168.)

$f(x-1) = \frac{1}{2} \ln \frac{x}{2-x}$

a) $x - 1 = v$

$f(v) = \frac{1}{2} \ln \frac{v+1}{1-v}$

$f(x) = \frac{1}{2} \ln \frac{x+1}{1-x}$

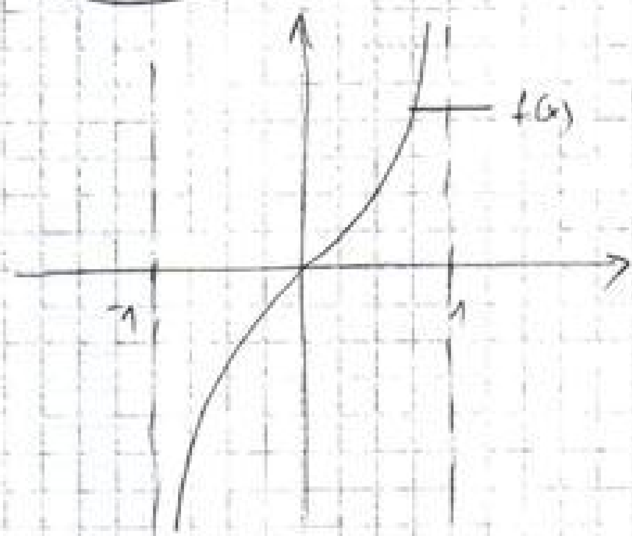
b)

b) $\frac{1+x}{1-x} > 0$

	$-\infty$	-1		1	$+\infty$
$1+x$	$-$	0	$+$		$+$
$1-x$	$+$		$+$	0	$-$
	$-$		$+$		$-$

$D(f) \in (-1, 1)$

GRAF ?



210.

$$\lim_{x\to 1} \frac{x^2 - \sqrt{x}}{\sqrt{x} - 1} = A$$

$\sqrt{x} = t \Rightarrow x^2 = t^4$

$$A = \lim_{t\to 1} \frac{t^4 - t}{t - 1} = \lim_{t\to 1} \frac{t(t^3 - 1)}{t - 1} =$$

$$= \lim_{t\to 1} \frac{t\cancel{(t-1)}(t^2+t+1)}{\cancel{(t-1)}} = \lim_{t\to 1} t(t^2+t+1)$$

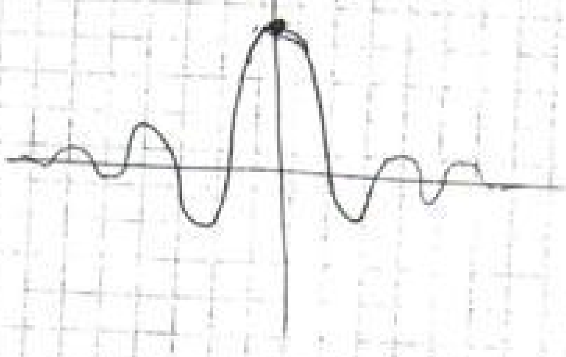
$= 1 \cdot 3 = 3$

Vježbe (PREKID)

1. VRSTE

$$f(x) = \frac{\sin x}{x}$$

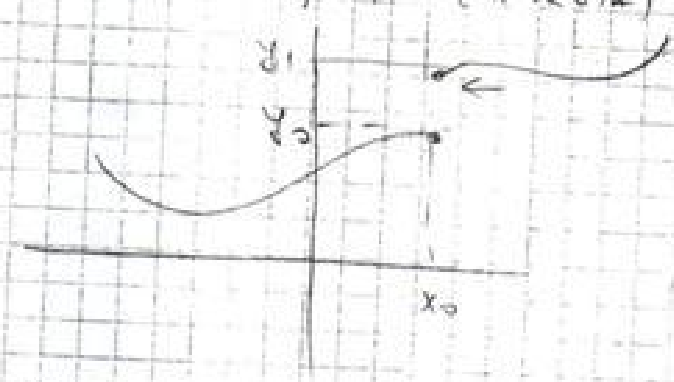
OTKLONJIVI PREKID (1. VRSTA)



$$\lim_{x \to 0-} f(x) = \lim_{x \to 0+} f(x) = L = 1$$

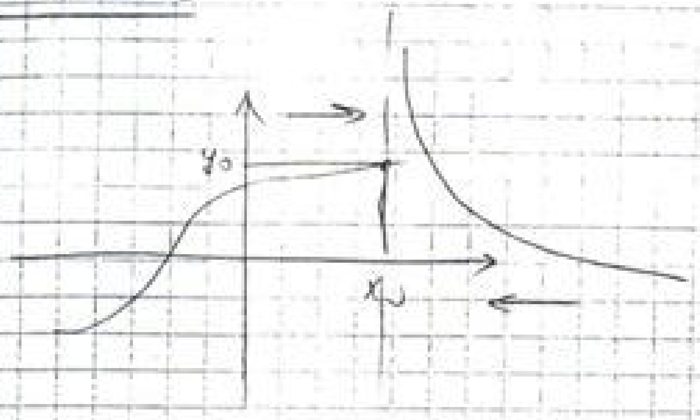
$$f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

NEOTKLONJIVI (1. VRSTA)



$$\left.\begin{array}{l} \lim_{x \to x_0+} f(x) = y_1 \\ \lim_{x \to x_0-} f(x) = y_0 \end{array}\right\} \neq$$

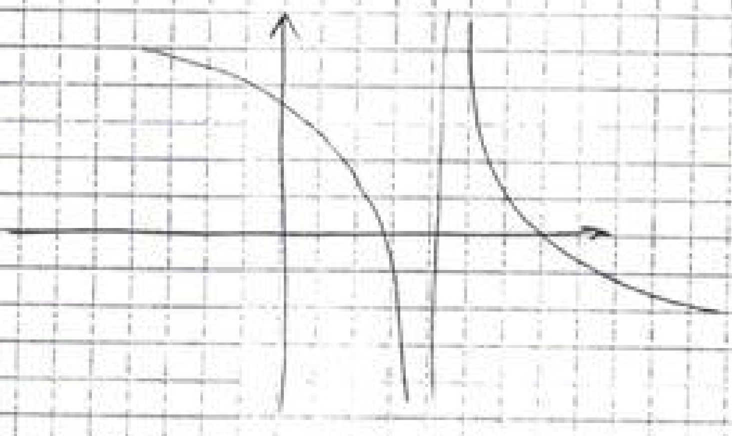
2. VRSTE

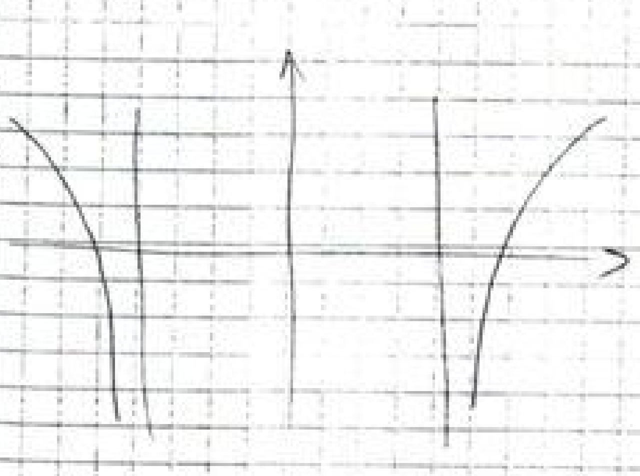


$\lim\limits_{x \to x_{0-}} (f(x) = y_0 \longrightarrow$

$\lim\limits_{x \to x_{0+}} f(x) = \pm\infty \longleftrightarrow$ asimptota

$\boxed{x = x_0}$ — V.A.





ASIMPTOTE

$$\lim_{x\to a\pm 0} f(x) = \pm\infty \Rightarrow x = a \quad V.A.$$

K.A. . . $y = kx + n$

$k = 0 \to$ H.A.

$$k = \lim_{x\to\pm\infty} \frac{f(x)}{x}$$

$$n = \lim_{x\to\pm\infty} \left[f(x) - kx \right]$$

H.A. . . $y = n$

$$x\, sine^y + sine^y = x$$

$$x(1 - sine^y) = sine^y$$

$$x = \frac{sine^y}{1 - sine^y}$$

$$y = \frac{sine^x}{1 - sine^x}$$

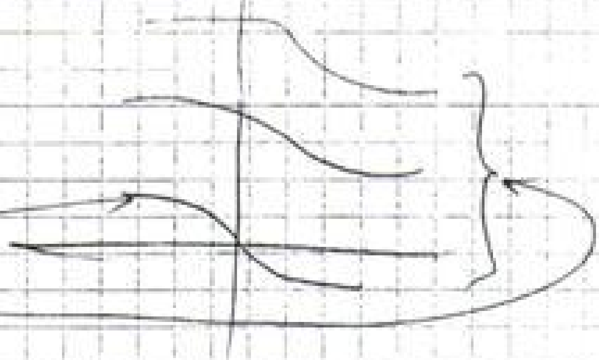
TUTORIJAL 7

(1717.) Nađi inverznu f.

$$y = \ln\left(\arcsin\frac{x}{x+1}\right)$$

arc – prva f.

Arc – čitava grana f.



- Ako je $f(x)$ monotona i neprekidna f. ⇒ ima inverznu.

D_f:

$$\land\ \begin{cases} \arcsin\frac{x}{x+1} > 0 \Rightarrow 0 < \frac{x}{x+1} \le 1 \\ \left|\frac{x}{x+1}\right| \le 1 \end{cases}$$

$\frac{x}{x+1} > 0 \qquad -\frac{1}{x+1} < 0$

$\land \rightarrow \boxed{x \in (0, \infty)}$

neprekidna

- Kompozicija monotonih f. je monotona f.

$$y = \ln x \circ \arcsin x \circ \underbrace{\frac{1}{x} \circ x+1}_{\frac{x}{x+1}}$$

$x \uparrow, \quad x+1 \uparrow, \quad \frac{1}{x+1} \downarrow, \quad \underbrace{1 - \frac{1}{x+1}}_{\frac{x}{x+1}} \uparrow$

$$y = \ln\left(\arcsin\frac{x}{x+1}\right)$$

$$y = \log_e\left(\arcsin\frac{x}{x+1}\right)$$

$$e^y = \arcsin\frac{x}{x+1} \qquad \sin e^y = t$$

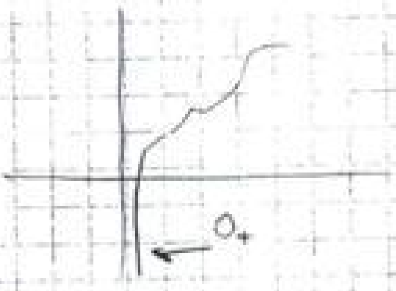
$$\sin e^y = \frac{x}{x+1} \,/\, x+1$$

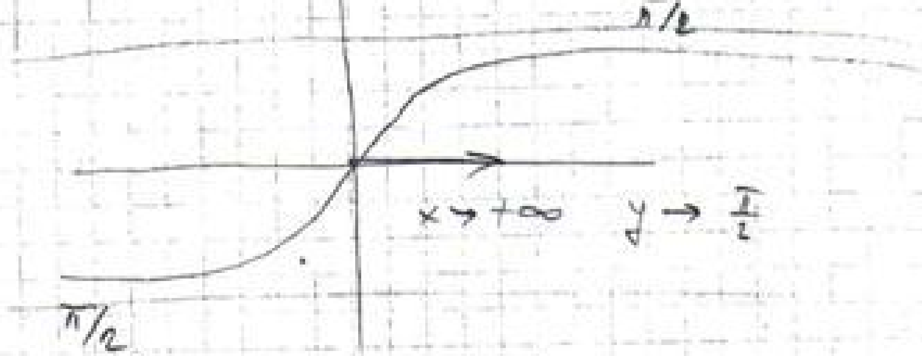
$\arcsin \frac{x}{x+1} \Uparrow$, $\ln\left(\arcsin \frac{x}{x+1}\right) \uparrow$

$M \uparrow$

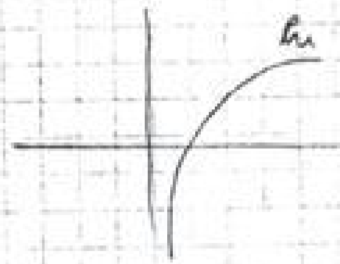
$f^{-1}(x)$

$\underline{R(f)}$: $y(0_+)$





$\pi/2$

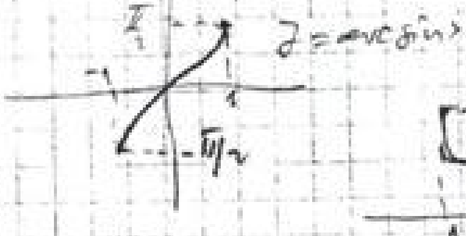


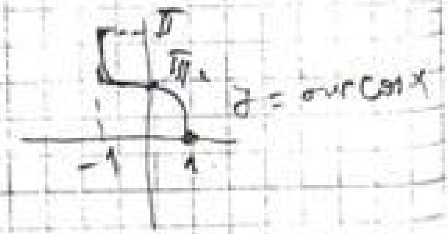
$y(0_+) = \lim\limits_{x \to 0_+} y(x) = \ln 0_+ = -\infty$

$y(\infty) = \lim\limits_{x \to +\infty} y(x) = \ln \frac{\pi}{2} \to$ H. A.

$\lim\limits_{x \to +\infty} \ln\left(\arcsin \frac{x}{x+1}\right)$

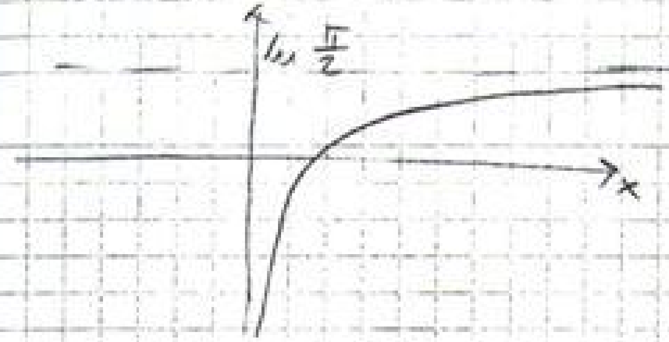
$\ln \arcsin\left(\lim\limits_{x \to \infty} \frac{x}{x+1}\right) = \ln \arcsin 1 = \ln \frac{\pi}{2}$





R f

$\boxed{-\infty < y < \ln \frac{\pi}{2}}$ f. ograničena



$x = y$

$x = \ln \arcsin \frac{y}{y+1}$

$e^x = \arcsin \frac{y}{y+1}$

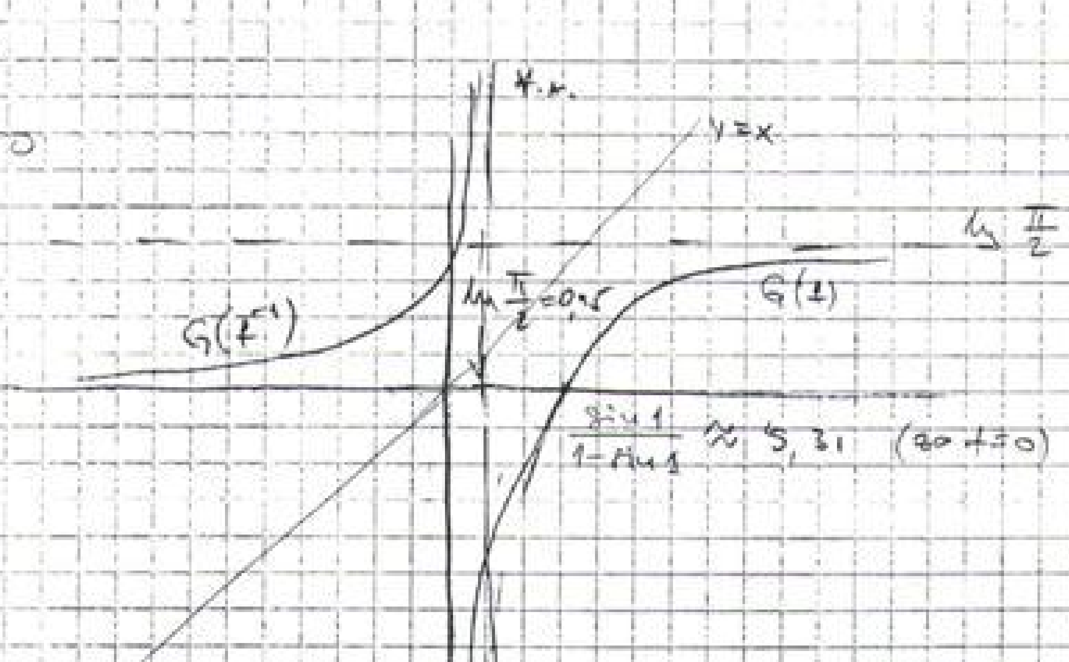
$\sin e^x = \frac{y}{y+1} \Rightarrow f^{-1}(x) = \frac{\sin e^x}{1 - \sin e^x}$

$\boxed{R f(x) = D f^{-1}(x)}$

$D f: x \in (0, +\infty)$

$x > 0$

$R f^{-1}: y > 0$



1723. Odrediti zavisnost između x i y

$$x = \sin t + \sqrt{3}\cos t$$
$$y = t + \sin(t + \underbrace{\operatorname{arc\,tg}\sqrt{3}}_{\frac{\pi}{3}})$$

$y = f(x) = ?$

$$x = 2\left(\underbrace{\tfrac{1}{2}}_{\cos\frac{\pi}{3}}\sin t + \underbrace{\tfrac{\sqrt{3}}{2}}_{\sin\frac{\pi}{3}}\cos t\right)$$

$$x = 2\sin\left(t + \frac{\pi}{3}\right)$$

$\sin\alpha = \sin\beta$

$\alpha = (-1)^n\beta + n\pi \quad (n \in \mathbb{Z})$

$t + \frac{\pi}{3} = (-1)^n \arcsin\frac{x}{2} + n\pi \quad (*)$

$t =$

$y = t + \underbrace{\sin\left(t + \frac{\pi}{3}\right)}_{\frac{x}{2}}$ $\qquad \sin\left(t + \frac{\pi}{3}\right) = \frac{x}{2} \quad (**)$

iz $(*)$ i $(**)$ $\Rightarrow$ $\boxed{y = \underbrace{(-1)^n \arcsin\frac{x}{2} + n\pi - \frac{\pi}{3}}_{t} + \underbrace{\frac{x}{2}}_{\frac{x}{2}}}$

$(n \in \mathbb{Z})$

$\left|\frac{x}{2}\right| \le 1 \qquad n = 0 - 1$ grana arcusa

$|x| \le 2$

$\boxed{n=0}$ prva grana rješavanje f.

Df: $|x| \leq 2$

$y = \arcsin \frac{x}{2} + \frac{x}{2} - \frac{\pi}{3}$

2044.

$f(x) = \frac{\sqrt{x+1} - 1}{\sqrt[3]{x+1} - 1}$ $\quad x \neq 0$

$x \geq -1$

- odrediti vrijednost da se otklonio prekid

$(x+1 \geq 0 \quad -1) \neq 0$

$x \in [-1, 0) \cup (0, +\infty)$ def. reg.

$L = \lim_{x \to 0} \frac{\sqrt{x+1} - \sqrt{1}}{\sqrt[3]{x+1} - \sqrt[3]{1}}$

$$\boxed{\sqrt{a} - \sqrt{b} = \frac{a-b}{\sqrt{a} + \sqrt{b}}}$$ $a > 0, \ b > 0$

$$\boxed{\sqrt[3]{a} - \sqrt[3]{b} = \frac{a-b}{\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2}}}$$

$L = \lim_{x \to 0} \frac{x+\cancel{1}-\cancel{1}}{\sqrt{x+1} + \sqrt{1}} \cdot \frac{\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + 1}{x+\cancel{1}-\cancel{1}}$

$L = \frac{3}{2}$

prošireno

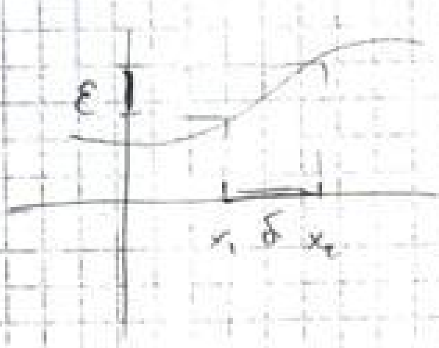
$$\tilde{f}(x) = \begin{cases} f(x), & x \neq 0 \\ \frac{3}{2}, & x = 0 \end{cases}$$

4. Ispitati uniformnu neprekidnost:

$$f(x) = \sin\frac{\pi}{x}, \quad x \in (0,1]$$

Uniformnost - stroga neprekidnost područje D

- Funkcija f je uniformno (ravnomjerno) neprekidna na datom skupu E_x ako je $f(x)$ def na E_x i $(\forall \varepsilon > 0) \Rightarrow (\exists \delta = \delta(\varepsilon) > 0): x_1, x_2 \in E_x$ uslov da se prirast arg može ograničiti $|x_2 - x_1| < \delta \Rightarrow |f(x_2) - f(x_1)| < \varepsilon$



nije uniformna

- Unif. nep. $\Longrightarrow$ obična nep.

$\not\Longleftarrow$

$f(1) = 0$

$\lim\limits_{x\to 0_+} f(x) = L$ nepostoji i nema produženja
(+ ogr. na argumentu)

$O(0_+)$: $x_1, x_2 \in O(0_+)$

$x_1 = \frac{1}{n}$ $\qquad x_2 = \frac{2}{2n-1}$

$|x_2 - x_1| < \delta$

$|x_2 - x_1| = \left|\frac{1}{n} - \frac{2}{2n-1}\right| = \left|\frac{1}{n(2n-1)}\right| \xrightarrow[\infty]{n\to\infty} 0$

ako je $\frac{1}{n(2n+1)} \not\to$ δ ograničeno

δ se može učiniti po volji malim

$\Delta = |f(x_2) - f(x_1)|$

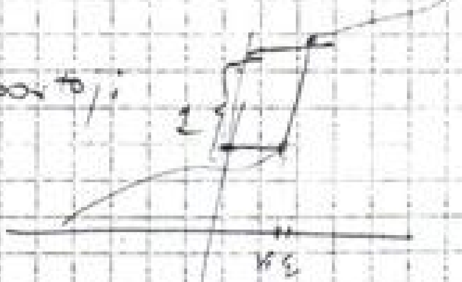
$= \left|\sin \pi n - \sin \frac{\pi(2n-1)}{2}\right|$

$= \left|\sin \pi n - \sin\left[\pi\left(n - \frac{1}{2}\right)\right]\right| = \left|(-1)^{n+1}\right| = 1 = \text{konst.}$

ne može se učiniti po volji malim

$|f(x_2) - f(x_1)| < \varepsilon \overset{n\to\infty}{\not\longrightarrow} 0$

- uniformna neprekidnost ne postoji



5.

$f(x) = \ln x$

$x_1 = \frac{1}{e^n}$

$x_2 = \frac{1}{e^{n-1}}$

$|x_2 - x_1| = \left| \frac{1}{e^{n-1}} - \frac{1}{e^n} \right| = \left| \frac{1 - e^{-1}}{e^n \cdot e^{-1}} \right| =$

TUTORIJAL 7

227.

$f(x) = x \ln x$

$a = 1$

$a = e$

1° $\Delta x = x - a = x - 1$

$\Delta y = f(x) - f(a) = f(x) - f(1) = f(x) = x \ln x$

2° $\Delta x = x - e = h$

$\Delta y = f(h+e) - f(e) = (e+h)\ln(e+h) - e \ln e^{1}$

$= (e+h)\ln(e+h) - e$

228.

a) $f(x) = 3x + 10$ neprekidna ?

$$\left(|x - x_0| < \delta\right) \Rightarrow \left(|f(x) - f(x_0)| < \varepsilon\right)$$

$|(3x+10) - (3x_0+10)| < \varepsilon$

$|3(x - x_0)| < \varepsilon$

$3|x - x_0| < \varepsilon$ $\qquad |x - x_0| < \delta$

$3\delta \leq \varepsilon$

$\delta \leq \frac{\varepsilon}{3}$

$\delta = \frac{\varepsilon}{3}$

$$\left(|x - x_0| < \delta\right) \Rightarrow \left(|f(x) - f(x_0)| = 3|x - x_0| < 3\delta = \varepsilon\right)$$

Za $\forall \varepsilon > 0$ možemo uzeti $\delta = \frac{\varepsilon}{3} \Rightarrow$ neprekidnost f.

u x_0

b) $f(x) = \frac{2}{x}$

$$\left(|x-x_0| < \delta\right) \Rightarrow \left(|f(x) - f(x_0)| < \varepsilon\right)$$

$$|f(x) - f(x_0)| = \left|\frac{2}{x} - \frac{2}{x_0}\right| = \frac{2|x_0 - x|}{x x_0} = \frac{2|x-x_0|}{x \cdot x_0}$$

$$|x-c| < \frac{1}{2}|c|$$

$$x \neq 0 \quad i \quad |x| > \frac{1}{2}|c|$$

$$|x \cdot c| = \frac{1}{2}|c| \cdot |c| = \frac{1}{2}|c|^2$$

$$\left(|x-c| < \frac{1}{2}|c|\right) \Rightarrow \left(|f(x) - f(x_0)| = \frac{2|x-x_0|}{x \cdot x_0} \leq \frac{4|x-x_0|}{|x_0|^2}\right.$$

$$\frac{4\delta}{|x_0|^2} \leq \varepsilon \Rightarrow 4\delta \leq \varepsilon |x_0|^2$$

$$\delta \leq \frac{\varepsilon |x_0|^2}{4}$$

$$\delta \min = \left\{ \frac{1}{2}|x_0|, \frac{|x_0|^2 \cdot \varepsilon}{4} \right\}$$

$$\delta < \frac{1}{2}|c|$$

$$|x-c| < \delta < \frac{|c|^2 \varepsilon}{4}$$

$$|f(x) - f(x)| < \frac{4}{|x_0|^2} \cdot \frac{|c|^2 \varepsilon}{4} = \varepsilon$$

$$\left.\begin{array}{l} \left|\frac{1}{n} - \frac{2}{2n-1}\right| = \left|\frac{1}{n(2n-1)}\right| \xrightarrow{n \to \infty} 0 \\ \left|\frac{\frac{2}{1}}{n} - \frac{\frac{2}{2}}{2n-1}\right| = |2n - 2n-1| = -1 \end{array}\right\} \text{nije uniformno}$$

a) $\left(|x-x_0|<\delta\right) \Rightarrow \left(|f(x)-f(x_0)|<\varepsilon\right)$

$$|f(x)-f(x_0)| = \left|\frac{6x}{|x|}-\frac{6x_0}{|x_0|}\right|$$

$$|x-x_0| < \frac{1}{2}|x_0|$$

- oba + ili oba -

$$\left(f(x)=6 \land f(x_0)=6\right) \lor \left(f(x)=-6 \land f(x_0)=-6\right)$$

$$f(x)=f(x_0)$$

za $\varepsilon>0$ doj $\delta=\frac{1}{2}|x_0|$

$$\left(|x-x_0|<\delta\right) \Rightarrow \left(|f(x)-f(x_0)|<\varepsilon\right)$$

$\boxed{c=0}$ $x>0$

$$|f(x)-f(0)| = \left|\frac{6x}{|x|}+6\right| = 12 \quad ?$$

234.

$$f(x)=\begin{cases} -2x+1 & \text{za } (-1\le x<0) \lor (0<x\le 1) \\ 2x+1 & \text{za } (1<x<2) \\ \dfrac{1}{x-3} & \text{za } (2\le x<+\infty) \end{cases}$$

1° $\boxed{x=1}$

$$\lim_{x\to 1_+} f(x) = \lim_{x\to 1_+}(2x+1)=3$$

$$\lim_{x\to 1_-} f(x) = \lim_{x\to 1_-}(-2x+1)=-1$$

$$\lim_{x\to 1_+} f(x) \neq \lim_{x\to 1_-} f(x)$$

$x=1 \rightarrow$ tačka prekida

$f(1-) = -1 = f(1) \rightarrow$ neprekidna s lijeva u $x=1$

$f(1+) = 3 \neq f(1) \rightarrow$ nije neprekidna s desna u $x=1$

3° $\boxed{x=0}$

$$\left.\begin{aligned} f(0_+) &= \lim_{x\to 0_+} f(x) = \lim_{x\to 0_+} (-2x+1) = 1 \\ f(0_-) &= \lim_{x\to 0_-} f(x) = \lim_{x\to 0_-} (-2x+1) = 1 \end{aligned}\right\} \lim_{x\to 0} f(x) = 1$$

$-\lim$ postoji u $x=0$ ali $x=0 \notin \mathcal{D}(f)$

nije neprekidna ni s lijeva ni s desna

– u $x=0$ otklonjiv prekid (1 vrste)

$$g(x) = \begin{cases} f(x) & x \in \mathcal{D}(f) \\ 1 & x=0 \end{cases}$$

$g(0_-) = g(0_+) = g(0) = 1$

5° $\boxed{x=-1}$

$f(-1_+) = \lim_{x\to -1_+} f(x) = \lim_{x\to -1_+} (-2x+1) = 3 = f(-1)$

– f neprekidna s desna u $x=-1$

– f. nije def. lijevo od -1

4° $\boxed{x=2}$

$f(2_+) = \lim_{x\to 2_+} f(x) = \lim_{x\to 2_+} \left(\frac{1}{x-3}\right) = -1$

$\lim_{x\to 2_-} f(x) = \lim_{x\to 2_-} (2x+1) = 5$ ne prekidna s lijeva desna

$\lim_{x\to 2_-} f(x) = 5 \neq \lim_{x\to 2_+} f(x) = -1$

prekid u $x=2_-$ s lijeva $= \lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{1}{x-3} = -1$

$f(2) = -1$

$\lim\limits_{x \to 2+} f(x) = \lim\limits_{x \to 2} f(x) = -1$

- neprekidna sa desna
- prekidna: skok (1. vrste - neodstranjiv)

$x = 3 \notin D(f)$

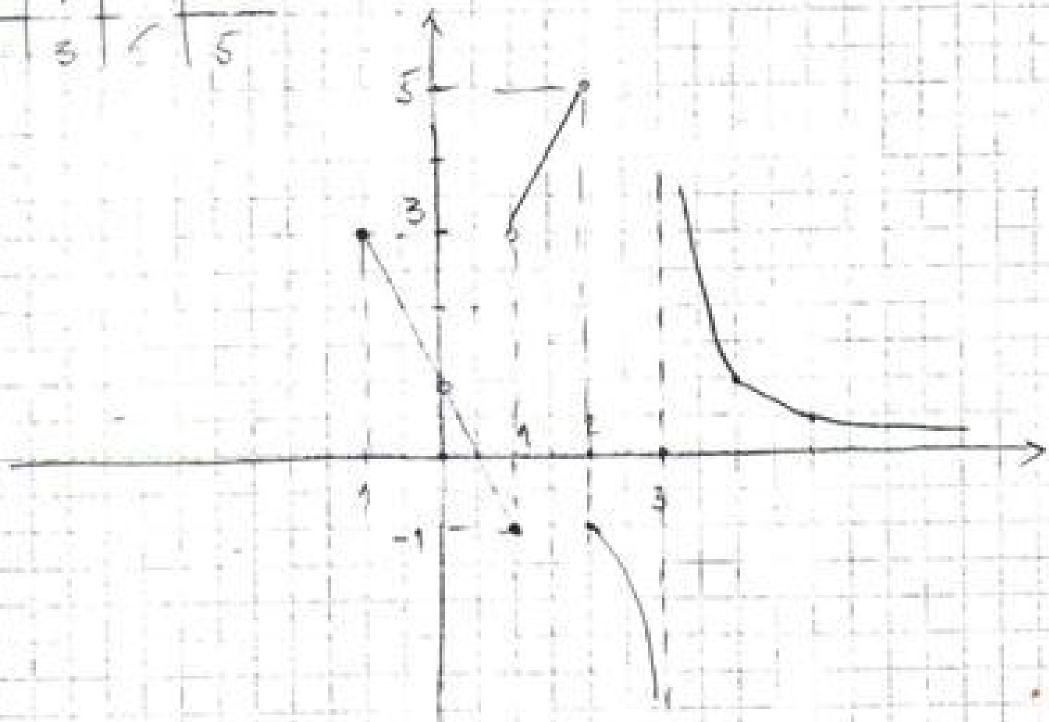
$\lim\limits_{x \to 3-} f(x) = \lim\limits_{x \to 3-} \left(\frac{1}{x-3}\right) = -\infty$

$\lim\limits_{x \to 3+} f(x) = \lim\limits_{x \to 3+} \left(\frac{1}{x-3}\right) = +\infty$

x	-1	0	1
$-2x+1$	3	1	-1

x	2	3	4	5
$\frac{1}{x-3}$	-1	0	1	$\frac{1}{2}$

x	1		
$2x+1$	3		5



235.

$$f(x)=\begin{cases} -2\cos x & za \quad x \le -\pi \\ a\cos x + b & za \quad -\pi < x < \pi \\ 2\sin\frac{x}{2} & za \quad x \ge \pi \end{cases}$$

$f: A \to R$

$f(c-) = f(c+) = f(c)$

$D(f) = R$

- neprekidna na intervalima

$(-\infty, -\pi)$; $(-\pi, \pi)$; $(\pi, +\infty)$

1) $\lim_{x\to -\pi_-} f(x) = \lim_{x\to -\pi_+} f(x) = \lim_{x\to \pi} f(x)$ $\quad x = -\pi$

$\Leftrightarrow \lim_{x\to -\pi_-} (-2\cos x) = \lim_{x\to -\pi_+} (a\cos x + b) = \lim_{x\to -\pi} (-2\cos(-\pi)) = 2$

$(a\cdot(-1) + b = 2) \Rightarrow -a+b=2$

2) $\lim_{x\to \pi_-} f(x) = \lim_{x\to \pi_+} f(x) = \lim_{x\to \pi} f(x)$

$\overbrace{\lim_{x\to \pi_-} (a\cos x + b)}^{-a+b} = \lim_{x\to \pi_+} (-2\cos x) = \lim_{x\to \pi} \left(2\sin\frac{\pi}{2}\right) = 2$

$(-a+b=2)$

(243)

$$f(x) = 2 + \frac{1}{1 + 2^{\frac{1}{1-x}}}$$

$D(f) = \mathbb{R} \setminus \{1\}$

$$f(1_-) = \lim_{x \to 1_-} f(x) = \lim_{y \to 0_+} \left(\frac{1}{1 + 2^{\frac{1}{1-(1-y)}}} + 2 \right) = 0 + 2 = 2$$

$$f(1_+) = \lim_{x \to 1_+} f(x) = \lim_{y \to 0_+} \left(\frac{1}{1 + 2^{\frac{1}{1-(1-y)}}} \right) = 1 + 2 = 3 \; ?$$

$f(1_-) \neq f(1_+) \rightarrow$ prelid 1. vrste

ADNAN – TUTORIJAL 7

VJEŽBANJE

1709. Naći funkciju inverznu funkciji:

$$y=\sqrt[3]{x+\sqrt{1+x^2}}+\sqrt[3]{x-\sqrt{1+x^2}} \quad /^3$$

$$y^3=x+\cancel{\sqrt{1+x^2}}+3\sqrt[3]{\left(x+\sqrt{1+x^2}\right)^2}\cdot\sqrt[3]{x-\sqrt{1+x^2}}+3\sqrt[3]{x-\sqrt{1+x^2}}\cdot$$

$$\cdot\sqrt[3]{\left(x+\sqrt{1+x^2}\right)^2}+x-\cancel{\sqrt{1+x^2}}=$$

$$y^3=2x+3\sqrt[3]{\left(x+\sqrt{1+x^2}\right)\left(x-\sqrt{1+x^2}\right)}\underbrace{\left(\sqrt[3]{x+\sqrt{1+x^2}}+\sqrt[3]{x-\sqrt{1+x^2}}\right)}_{y}$$

$$y^3=2x+3\sqrt[3]{x^2-1-x^2}\cdot y$$

$$y^3=2x+3\sqrt[3]{-1}\,y$$

$$y^3=2x-3y$$

$$2x=y^3+3y$$

$$x=\frac{y^3+3y}{2}$$

$$y^{-1}=\frac{x^3+3x}{2}$$

1711. Naći inverznu funkciju funkcije:

$$y=\ln\arcsin\frac{x}{x+1}$$

$$e^y=\arcsin\frac{x}{x+1}$$

$$\sin e^y=\frac{x}{x+1} \quad /\cdot(x+1)$$

$$x\sin e^y+\sin e^y=x$$

$x - x \sin e^y = \sin e^y$

$x(1 - \sin e^y) = \sin e^y$

$y' = \dfrac{\sin e^x}{1 - \sin e^x}$

(1722) U sljedećim zadacima eliminirati parametar t i naći zavisnost između x i y.

$x = \sin t + \sqrt{3}\cos t$ $\qquad tg\frac{\pi}{3} = \sqrt{3}$

$y = t + \sin(t + arc\,tg\sqrt{3})$.

- Stavimo na adicionu formulu

$x = \sin t + tg\frac{\pi}{3}\cos t$

$x = \sin t + \dfrac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}\cos t \quad / \cos\frac{\pi}{3}$

$\frac{1}{2}$

$\cos\frac{\pi}{3}\, x = \sin t \cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos t$

$\frac{x}{2} = \sin\left(t + \frac{\pi}{3}\right) \qquad (1)$

$y = t + \sin\left(t + \frac{\pi}{3}\right) \qquad (2)$

---

(1) u (2)

$y = t + \frac{x}{2} \qquad (2')$

iz (1) $\quad t + \frac{\pi}{3} = arc\sin\frac{x}{2}$

$t = arc\sin\frac{x}{2} - \frac{\pi}{3}$ u $(2')$

$$\boxed{y = arc\sin\frac{x}{2} - \frac{\pi}{3} + \frac{x}{2}}$$

(1757.) Nacrtati grafik funkcije

$$y=\frac{x^2}{x+1}$$

1° $D(f) \in R \setminus \{x+1 \neq 0\} = R \setminus \{-1\}$

2° Pošto domen funkcije $D(f)$ nije simetričan u odnosu na koordinatni početak, nema smisla ispitivati parnost i neparnost funkcije

3° Nule funkcije

$\frac{x^2}{x+1}=0$ za $x=0$

4° Znak

$\frac{x^2}{x+1} \gtrless 0$

$y>0$ za $\forall x \in (-1,0) \cup (0,+\infty)$

$y<0$ za $\forall x \in (-\infty,-1)$

	$-\infty$		-1		0		$+\infty$
x^2		+		+	0	+	
$x+1$		−	0	+		+	
		−		+		+	

5° asimptote

H.A. ... $\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{x^2}{x+1} = \lim_{x\to\pm\infty} \frac{x}{1+\frac{1}{x}} = \frac{\infty}{1} = \infty$

V.A. ... $\lim_{x\to -1_-} f(x) = \lim_{x\to -1_-} \frac{x^2}{x+1} = \frac{1}{0_-} = -\infty$

$\lim_{x\to -1_+} f(x) = \lim_{x\to -1_+} \frac{x^2}{x+1} = \frac{1}{0_+} = +\infty$

V.A. ... $x=-1$

k 1. $y = kx + n$

$$k = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{\frac{x^2}{x+1}}{x} = \lim_{x\to\pm\infty} \frac{x^2}{x^2+1} = 1$$

$$n = \lim_{x\to\pm\infty} \left[f(x) - kx \right] = \lim_{x\to\pm\infty} \frac{\cancel{x^2} - \cancel{x^2} - x}{x+1} =$$

$$= \lim_{x\to\pm\infty} \frac{-x}{x+1} = -1$$

$y = x - 1$

6° tok

$$y' = \frac{2x(x+1) - x^2}{(x+1)^2} = \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$$

k.T. y' : $0, -2, -1$

	$-\infty$	-2	-1	0	$+\infty$
x	$-$	$-$	$-$	0 $+$	
$x+2$	$-$	0 $+$	$+$	$+$	
$(x+1)^2$	$+$	$+$	0 $+$	$+$	
	$+$	$-$	$-$	$+$	
y'	↗	↘	↘	↗	

7° ekstrem

$$y = \frac{x^2}{x+1}$$

$$y' = \frac{x(x+2)}{(x+1)^2} \qquad y' = 0 \Rightarrow \frac{x(x+2)}{(x+1)^2} = 0 \Rightarrow (x = 0 \ \lor \ x = -2)$$

$$y'' = \frac{(2x+2)(x+1)^2 - (x^2+2x)\,2\,(x+1)\cdot 1}{(x+1)^4}$$

$$= \frac{\cancel{2x^3} + \cancel{2x^2} + \cancel{4x^2} + \cancel{4x} + 2x + 2 - \cancel{2x^3} - \cancel{4x^2} - \cancel{2x^2} - \cancel{4x}}{(x+1)^4}$$

$$= \frac{2(x+1)}{(x+1)^4}$$

$$y''(0) = \frac{2(0+1)}{(0+1)^4} = \frac{2}{1} = 2 > 0 \Rightarrow \min$$

$$y''(-2) = \frac{2(-2+1)}{(-2+1)^4} = \frac{-2}{1} = -1 < 0 \Rightarrow \max$$

$$y(0) = \frac{0^2}{1+x} = 0 \Rightarrow A_{min}(0,0) \cup$$

$$y(-2) = \frac{(-2)^2}{1-2} = \frac{4}{-1} = -4 \Rightarrow B_{max}(-2,-4) \cap$$

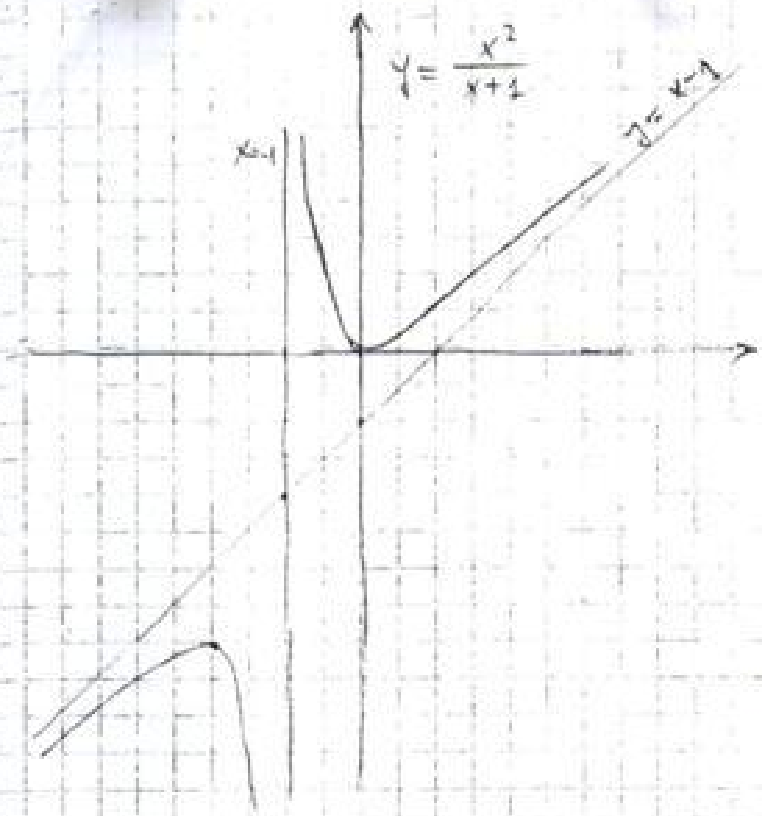
3° konveksitet

$$y'' = 0$$

$$\frac{2(x+1)}{(x+1)^4} = 0 \Rightarrow x = -1$$

$y \cap$ za $\forall x \in (-\infty, -1)$

$y \cap$ za $\forall x \in (-1, -\infty$

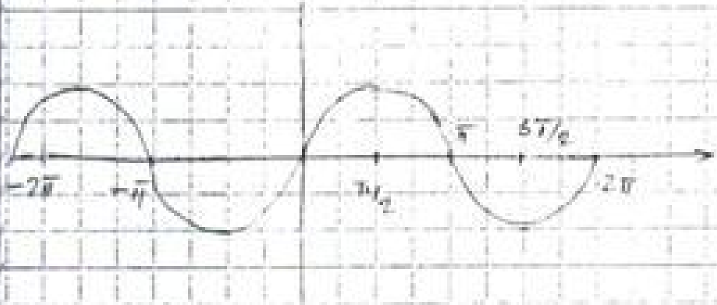


$y = \frac{x^2}{x+1}$
$y = x-1$
$x=-1$

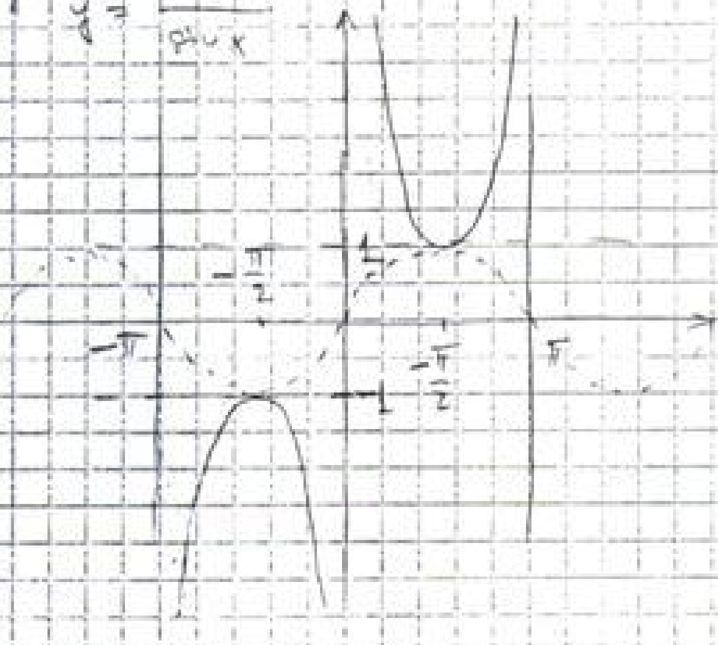
UŠĆUMLIĆ

17.54.

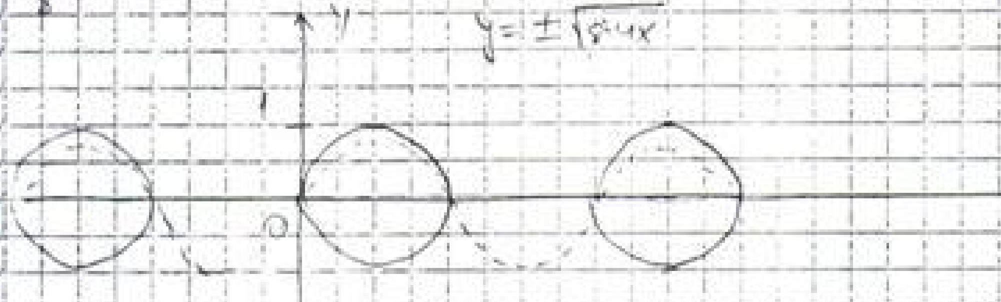
$\boxed{y = \sin x}$



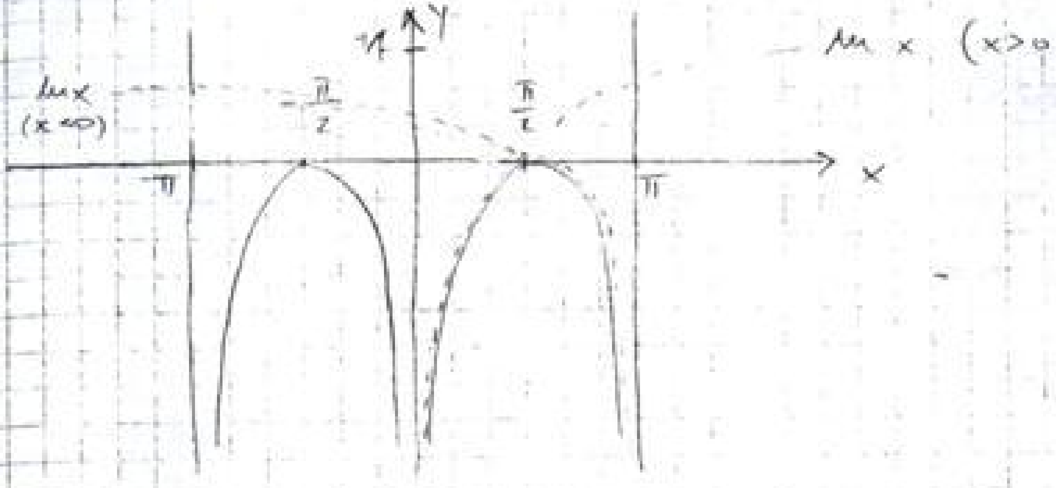
1° $y = \frac{1}{\sin x}$



2° $y = \pm\sqrt{\sin x}$

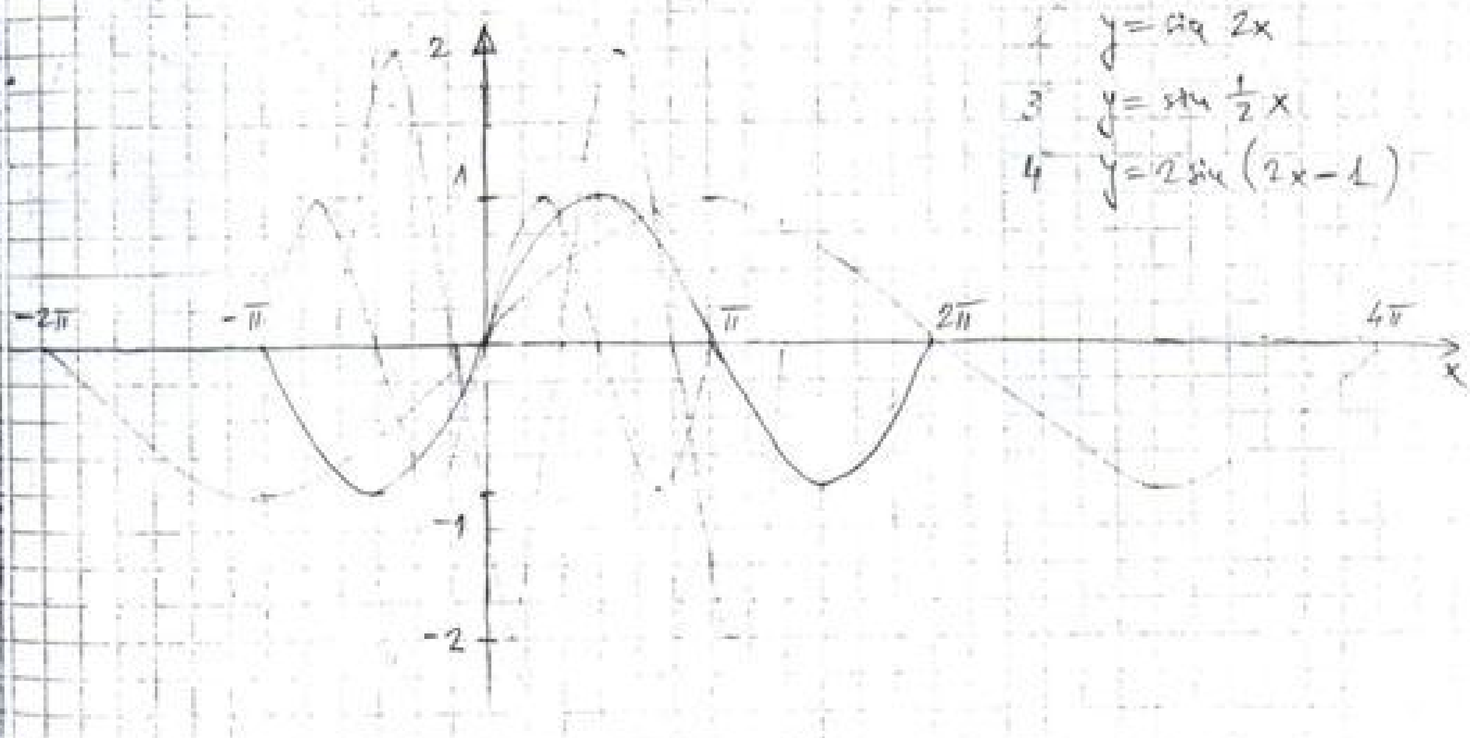


7° $y = \ln|\sin x|$



17.67.

1° $y = \sin x$
2° $y = \sin 2x$
3° $y = \sin \frac{1}{2} x$
4° $y = 2\sin(2x-1)$



(2023)

$$\lim_{x\to 0} \frac{2x^3 - 2x^2 + 2\sin x + 8x^6}{3x^5 + \sin^2 x + 4x - 7x^8}$$

- U brojniku i nazivniku imamo jako male veličine i ako $x \to 0$ sin teži 0, pa ih mogu zanemarit.

$$\lim_{x\to 0} \frac{2\sin x}{4x} = \frac{1}{2}$$

$\ln(1+x) \sim \frac{1}{x} \to 0$

$\lim \frac{2\alpha}{\alpha}$

(2024)

$$\lim_{\alpha\to 0} \frac{\ln(1+2\alpha-\alpha^2+\alpha^4-5\alpha^5)}{\ln(1+\alpha+2\alpha^2+\alpha^4)} = \lim_{\alpha\to 0} \frac{\ln(1+2\alpha)}{\ln(1+\alpha)} = 2$$

$\lim \frac{\ln(1+x)}{x}$

(2031)

$$\lim_{\alpha\to 0} \frac{\ln(1+\sin\alpha)}{\alpha} = 1 \qquad \ln \frac{1+2\alpha}{1+\alpha}$$

$e^{\frac{x}{x}}$

$1+\sin\alpha \sim \alpha$

$$\lim_{\alpha\to 0} \frac{\sin\alpha}{\alpha} = 1$$

lim

(2032) Dokazat uniformnost

$y = a^x \quad (a>0, \ \forall x)$

?

$|x_2 - x_1| < \delta \Rightarrow |f(x_2) - f(x_1)| < \varepsilon$

$x_1 = \frac{1}{n}$

$|x_2 - x_1| \to 0$

$x_2 = \frac{2}{2n-1}$

$$|f(x_2) - f(x_1)| = |a^{\frac{2}{2n-1}} - a^{\frac{1}{n}}$$

$$a^{\frac{1}{n}} - a^{\frac{1}{n-1}}$$

$$\frac{1}{n} - \frac{1}{n-1} = \frac{1}{n(n-1)}$$

2941.

$$y = \begin{cases} 1 - x^2 & , x < 0 \\ a & , x = 0 \\ 1 + x & , x > 0 \end{cases}$$

1° $x = 0$

$$\lim_{x \to 0_+} f(x) = \lim_{x \to 0_+} (1 + x) = 1$$

$$\lim_{x \to 0_-} f(x) = \lim_{x \to 0_-} (1 - x^2) = 1$$

$$\lim_{x \to 0_-} f(x) = \lim_{x \to 0_+} f(x) = \lim_{x \to 0} f(x) = \lim a = 1$$

$$\boxed{a = 1}$$

$y = 1 - x^2$ $\qquad$ $y = a = 1$ $\qquad$ $y = 1 + x$

x	-2	-1	0
y	-3	0	1

x	0	1	2
y	1	2	3